Credit Supply Driven Boom-Bust Cycles*

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Abstract

We develop a general equilibrium model capturing interactions among household, firm, and bank balance sheets. Our model reveals that credit supply changes, triggered by exogenous shocks to bank leverage and amplified by the endogenous deterioration of bank balance sheets during the bust, significantly contributed to the U.S. boom-bust cycle around 2008. The model generates changes in household mortgage credit during the boom, consistent with the micro-level evidence, some of which are argued to be inconsistent with the credit supply mechanism. Overall, our results underscore the critical role of credit supply in shaping the boom-bust cycle.

JEL Codes: E21, E32, E44, E60, G20, G51.
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1 Introduction

The housing market in the US (and in many other countries) experienced a dramatic boom-bust cycle during the last two decades. Real house prices increased by more than 30 percent between 1995 and 2006, and then dropped by a similar amount between 2006 and 2011. Such a large decline in house prices pushed many homeowners with mortgages into negative equity, which then increased foreclosures significantly. Not only the housing market but also the financial sector and the rest of the macroeconomy struggled: the losses in mortgage related assets weakened bank balance sheets and concerns about the value of these assets made creditors withdraw from the wholesale funding market, disrupting the credit flow to non-financial firms and households.

How important were the changes in credit supply during the boom-bust cycle in the economy, as observed in the US around 2008? The literature presents two opposing views. On the one hand, Mian and Sufi (2009) have argued that the expansion of credit supply to marginal borrowers (interpreted as low income households at the time) was the root cause of the housing boom. On the theoretical front, Kiyotaki et al. (2011) have found that a decline in world interest rates could generate a large boom in house prices. Similarly, Justiniano et al. (2017) have posited that the increase in credit supply, driven by looser lending constraints in the mortgage market, by reducing borrowing rates, contributed to the housing boom.

More recent literature, on the other hand, with more detailed data and more developed models, favors the view that shifts in credit demand, driven by higher house price growth expectations, were the primary force behind the boom-bust cycle, leaving little role for credit supply. Kaplan et al. (2020) argued that the absence of a rental market and/or long-term defaultable mortgages was crucial for generating significant effects of credit supply on house prices in Kiyotaki et al. (2011), Justiniano et al. (2017) and similar papers. On the empirical front, Adelino et al. (2016) and Foote et al. (2016) found that credit growth was similar across different income groups, i.e., not stronger for the subprime borrowers, challenging the credit supply mechanism in Mian and Sufi (2009). Even though Mian and Sufi (2017) argued that the credit supply perspective on the mortgage boom can apply to the whole population rather than the lowest income individuals, testing this argument demands modeling the rich heterogeneity among households and analyzing their response to credit supply expansion.

In this paper, we reevaluate the credit supply channel by developing a model with an overlapping-generations structure of households who face idiosyncratic income risk under incomplete markets and choose between owning and renting a house of their desired size. Households can use long-term mortgages for their purchases and have the option to refinance, and default in any period throughout the life of the mortgage. Mortgage contracts internalize the default probabilities of households, therefore, each mortgage is individual-specific, and borrowing limits endogenously arise via limited commitment by households. With these features, the household sector in our model is very similar to the model studied by Kaplan et al. (2020), which allows us to address criticisms and empirical evidence against the credit supply channel.

The key innovation of our paper is to incorporate the interactions among household, firm, and bank balance sheets that, as we demonstrate, play a critical role in the boom-bust. Banks fund themselves through international investors and household deposits and issue short-term loans to firms and long-term mortgages to households. We assume that bankers can steal a fraction of assets and default as in Gertler and Kiyotaki (2010a). To avoid such behavior in equilibrium, lenders limit their funding to banks, creating an endogenous constraint on bank leverage. As a result, banks' ability to intermediate funds depends on their capital.

Firms finance part of their wage bill (working capital) through short-term loans from banks, making their labor and production decisions depend on the equilibrium bank lending rate, which critically depends on bank credit supply.[1]

To study the role of shifts in the credit supply during the boom-bust episode, we assume that the economy is initially in the steady state and calibrate the model to match several US data moments—most importantly, regarding household and bank balance sheets—before 1998. We then give two subsequent unexpected leverage shocks to bank balance sheets. First, in 1998, banks start increasing their leverage gradually over time. Second, in 2008, the leverage constraint reverts to its initial steady-state level. We calibrate the size and duration

[1]Christiano and Eichenbaum (1992), Cooley and Quadrini (1999, 2004), Neumeyer and Perri (2005), Mendoza (2010), and Jermann and Quadrini (2012) also use the working capital requirement to link the firms' production decisions to the borrowing rate.

of the boom shock such that the changes in the banks' book leverage and the credit spread to match the data during the boom and study the transition of our model economy in response to these shocks. Even if not targeted, the model generates a decline in bank leverage and a spike in credit spread at the time of the bust that are consistent with the data as well.

The main driver of the boom-bust cycle is the changes in the equilibrium bank lending rate in response to the credit supply shocks. With two unexpected and offsetting permanent shifts in bank leverage, the bank lending rate first decreases gradually and then unexpectedly reverts to its initial steady-state level after a big jump in 2008 due to a sharp deterioration of bank balance sheets.[2]

The benchmark economy, driven by credit supply shocks, generates a significant boom-bust cycle in the housing market and the macroeconomy, along with a slow recovery from the bust. In particular, credit supply shocks account for about 50 percent of the rise in house prices and 75 percent of the increase in the mortgage debt-to-GDP ratio. During the bust, the contraction of credit accounts for an even larger fraction of the decline in house prices (83 percent) and about 60 percent of the increase in foreclosure rates. On the real side of the economy, the credit supply shock accounts for about 85 and 92 percent of the booms in GDP and consumption, respectively, and and even larger portions of the busts in these variables.

The model's cross-sectional implications are also consistent with the recent evidence from detailed micro-level data analysis, some of which are argued to be inconsistent with the credit supply mechanism. In particular, our model generates uniform credit growth across different income quantiles over the boom episode, as shown to be the case in the data (Adelino et al. (2016), Foote et al. (2016), and Albanesi et al. (2017)). Since the credit supply shock does not only decrease the borrowing cost to households but also to firms, the combination of

[2]Even if we have chosen changes in bank leverage as the source of credit supply changes, the specific source is not crucial. What matters is that the increase in credit supply lowers the cost of borrowing (interest rate) for households and firms. The relaxation of lenders' constraints, as in Justiniano et al. (2017), or the saving glut of the rich, as in Mian et al. (2020), would generate a similar increase in credit supply and, consequently, reduce borrowing costs for households and firms, as in our model. Alternatively, one could select a decline in world interest rates, as in Kiyotaki et al. (2011), as the driving force. We have chosen the changes in bank leverage as the primary source because the banking sector was at the center of the boom-bust, especially during the 2008 financial crisis. Importantly, as we demonstrate, the interaction between household, bank, and firm balance sheets significantly amplifies the boom-bust.

lower borrowing costs as well as higher labor income during boom increase the demand for housing and mortgage debt of all households, not only the poor. Consistent with the findings of Adelino et al. (2016), our model implies that credit growth has been stronger for consumers with faster income growth. We also find that higher leverage and lower liquid asset holdings at the peak of the boom, as well as income loss during the bust, are the major factors contributing to increased foreclosures. These findings are consistent with the evidence presented in Gerardi et al. (2008), Foote et al. (2010), Palmer (2015), and Indarte (2023). The heterogeneous household structure in our model allows us to demonstrate that credit supply shocks not only generate a boom-bust cycle in aggregates consistent with the data, but also have micro-level implications in line with the micro evidence, strengthening the argument for the importance of the credit supply channel.

Having demonstrated that the credit supply channel can account for both the aggregate and distributional changes during the U.S. boom-bust cycle, we examine the drivers of the boom-bust cycle. Since exogenous shocks to bank leverage translate into changes in bank credit supply, which, in turn, affect the equilibrium bank lending rate, we start by exploring the effect of the changes in bank lending rate. The changes in the bank lending rate affect consumption and house prices *directly* through household borrowing costs but also *indirectly* through firm borrowing costs and, hence, household labor income. Our analysis reveals that the sizes of direct and indirect effects are comparable for house prices; however, the indirect effect is far more important for consumption since the change in consumption is driven by changes in both house prices and household labor income.

Second, we quantify the importance of the endogenous bank balance sheet deterioration during the bust, which manifests its effect through the spike in the bank lending rate. Two mechanisms, reinforcing each other, drive the deterioration of bank balance sheets: (i) changes in mortgage valuations and (ii) foreclosures. First, when banks reduce credit supply due to tightened leverage constraints, the equilibrium bank lending rate rises. Since mortgages are long-term assets, mortgage valuations decline, thereby worsening bank balance sheets. Consequently, banks further curtail credit, leading to a further increase in the bank lending rate. Second, as both house prices and labor income decline, a significant portion of mortgage borrowers default. As a result, bank balance sheets worsen because of the rise in foreclosures. We find that the valuation losses account for around 90 percent of the decline in bank net worth at the time of the bust, while the increase in foreclosures accounts for the rest, which is consistent with the evidence presented in IMF (2009).[3] Together, these two endogenous mechanisms cause a large decline in bank net worth, which amplifies the declines in output, house prices, and consumption by 61, 35, and 43 percent, respectively. These results reveal that the bank balance sheet deterioration amplifies the bust in variables that depend on shorter-term debt, such as output, but it has a relatively smaller effect on house prices, which depend on long-term debt, and a somewhat intermediate effect on consumption, which is driven by both house prices and labor income.

Finally, we compare the model's dynamics across credit supply, productivity, and housing demand shocks. While we find many similarities, there are also several important differences. For example, with housing demand shocks, households reduce capital accumulation, and thus output and labor income do not rise during the boom, and consumption rises only slightly. Another major difference is that the equilibrium bank lending rate does not increase significantly during busts with productivity and housing demand shocks. This is because, in contrast to credit supply shocks, these shocks primarily reduce credit demand, which mitigates the rise in the bank lending rate. As a result, relative to credit supply shocks, mortgage valuations and, hence, bank net worth decline by significantly less.

In summary, credit supply shocks can account for substantial portion of the boom-bust dynamics in aggregate variables, thanks to the connections between household, bank, and firm balance sheets. Moreover, the heterogeneity in the household sector allows us to demonstrate that the credit supply shock generates credit and foreclosure dynamics consistent with micro evidence–previously considered to be against the credit supply channel.

[3]Since we are exclusively decomposing the effect of the credit supply shock, this conclusion specifically pertains to the credit-supply-driven bust, which generates 60 percent of the increase in foreclosures. However, this does not imply that foreclosures accounted for only 10 percent of bank balance sheet losses, as the foreclosure rate was higher during the actual crisis.

Related Literature

Our paper contributes to the literature that studies the dynamics of the housing market and the macroeconomy around the 2008 financial crisis.[4] Using a model of representative borrower and saver, Justiniano et al. (2017) demonstrate that credit supply, driven by looser lending constraints in the mortgage market, accounts for the unprecedented rise in home prices, the surge in household debt, the stability of debt relative to home values, and the fall in mortgage rates.[5] However, Kaplan et al. (2020) argue that the absence of the rental market and/or long-term defaultable mortgages are critical for obtaining large effects of credit supply or credit conditions on house prices since, with rental markets, households can rent a house of their desired size if they are constrained in purchasing one. With these extensions, Kaplan et al. (2020) argue that shifts in household demand due to shocks to house price expectations, rather than changes in credit supply or conditions, were the main driving force behind the boom-bust cycle in the housing market.[6] They also find that temporary shocks to interest rate, essentially a credit supply shock, does not move house prices.

Credit supply expansion in our model is very similar to that in Justiniano et al. (2017). The relaxation of lending constraints of lenders in their case and the relaxation of leverage constraints of banks in our case lead to an increase in credit supply and a reduction in borrowing rates. However, we model the detailed household structure, as Kaplan et al. (2020) do, and yet still find a significant role for credit supply due to two main differences from the experiment conducted in Kaplan et al. (2020).[7] First, we consider permanent changes in bank leverage (and thus the bank lending rate) rather than factors like LTV, PTI, or temporary interest rate shocks. Second, the credit supply shock in our framework

[4]For surveys, see Davis and Van Nieuwerburgh (2015), Piazzesi and Schneider (2016), and Guerrieri and Uhlig (2016). See also Gomes et al. (2021) for a general survey of household financing decisions, including portfolio and mortgage choices.

[5]In a similar vein, Kiyotaki et al. (2011) and Adam et al. (2012) find that the decline in interest rates contributed substantially to the house price boom in the U.S. On the other hand, Greenwald (2016), using representative borrower and savers, and Huo and Rios-Rull (2013), Sommer et al. (2013), and Favilukis et al. (2017), using heterogeneous agent frameworks, show that changes in maximum LTV or payment-to-income (PTI) ratios can generate significant changes in house prices and consumption.

[6]Kiyotaki et al. (2011) also find that changes in LTV constraints do not affect house prices.

[7]We should note that the expansion of credit supply accounts for about 50 percent of the boom in house price, leaving the remaining 50 percent to be accounted for, plausibly by house price expectation shocks as in Kaplan et al. (2020).

does not solely affect households. The changes in credit supply, resulting from exogenous shocks to bank leverage and further amplified in the bust due to deterioration of bank balance sheets, lead to changes in the bank lending rate. This affects households directly by influencing their borrowing costs and indirectly through changes in firm borrowing costs. As a result, permanent changes in the bank lending rate generate significant income and wealth effects on households, giving rise to boom-bust cycles in the housing market and the broader macroeconomy.

Landvoigt (2016) and Diamond and Landvoigt (2022) also combine banking and household sectors to study the role of credit supply for the boom-bust in house prices and mortgage debt. Different from Landvoigt (2016) and Diamond and Landvoigt (2022), we model the feedback from bank balance sheets to firm borrowing, which significantly contributes to the boom-bust. Furthermore, the richer heterogeneity in our household sector allows us to compare our model's implications with micro data, that were argued to be against the credit supply channel.

Mechanisms in our model are supported by empirical findings. First, with detailed data from periods after 1996, Fraisse et al. (2020), Gropp et al. (2019), Aiyar et al. (2014), Jiménez et al. (2017), Gete and Reher (2018), and De Marco et al. (2021) causally link regulatory tightenings to declines in credit, and contractions in economic activity. Second, Gilchrist and Zakrajšek (2012a) and Gertler and Gilchrist (2018) show that credit spreads spike during downturns, predicting declines in subsequent economic activity. The credit spread dynamics in our model are similar to the Excess Bond Premium (EBP) dynamics reported in these papers. Third, Glaeser et al. (2012) and Justiniano et al. (2017) find that interest rates on firm loans and mortgages declined during the boom. Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, Ivashina and Scharfstein (2010) document a more than 50 percent decline in capital expenditure and working capital loans to corporations. Similarly, Adrian et al. (2013) find that capital expenditure and working capital loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis.

Our framework combines key elements from two strands of literature. On the one hand, an active literature has studied the pricing of default risk in the context of unsecured or mortgage debt. Prominent examples are Chatterjee et al. (2007), Livshits et al. (2010, 2007), Jeske et al. (2013), Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), Arslan et al. (2015), Guler (2015), Hatchondo et al. (2015), and Gete and Zecchetto (2023). In this literature, banks are often modeled as risk-neutral, zero-profit-making competitive financial intermediaries and has no balance sheet considerations, which we find to significantly amplify the bust.

On the other hand, the literature on bank balance sheets has examined how the depletion of a bank's capital reduces its ability to intermediate funds. This literature includes Mendoza and Quadrini (2010), Gertler and Kiyotaki (2010b, 2015), Gertler and Karadi (2011), Gertler et al. (2012), He and Krishnamurthy (2012, 2013), Brunnermeier and Sannikov (2014), Corbae and D'Erasmo (2013, 2019), and Bianchi and Bigio (2022). However, in this literature, banks' asset structures typically take a simple form, such as one-period bonds, or lack the rich heterogeneity observed in banks' portfolios. By modeling the rich heterogeneity of the household sector, we show that credit-supply shocks do not only generate a boom bust in aggregates, but also micro dynamics consistent with evidence.

2 Quantitative Model

The model economy is composed of five sectors: (i) households, (ii) financial intermediaries (banks), (iii) rental companies, (iv) firms, and (v) the government. Total housing stock is fixed at $\bar{H}$, but the homeownership rate is not. This becomes possible as part of the housing stock is owned by homeowners and the rest is owned by rental companies who rent it to the households. There is perfect competition in all markets.

There is no aggregate uncertainty. Boom-bust transitions are generated by two unexpected shocks, both perceived as permanent. Other than the shock periods, there is perfect foresight.[8] Since households are ex post heterogeneous, all the endogenous prices, value func-

[8] Solving this model under aggregate uncertainty is a formidable challenge, as there are many markets and prices to be determined in equilibrium, and the model is highly nonlinear. Therefore, we have chosen

tions, and policy functions depend on the aggregate state of the economy and the distribution of households. For notational convenience, we suppress these dependencies.

2.1 Households

We assume that households work until the mandatory retirement age J_r and live up to age J after the retirement. Households receive utility from consumption and housing services and can choose between renting and owning a house of their desired size. Household preferences take the following form:

$$E_0 \sum_{j=1}^{J} \beta^{j-1} u\left(c_j, s_j\right),$$

where E_0 is the expectations operator, β is the discount factor, c_j is consumption, and s_j is the housing services at age j.

Working-age households are subject to idiosyncratic income uncertainty: before retirement, labor income consists of a deterministic component $f(j)$, which only depends on age, and a stochastic component z_j, which is an AR(1) process, and an aggregate component w. Thus, a household's labor income process $y(j, z_j)$ can be summarized by

$$y(j, z_j) = \begin{cases} (1-\tau)\, w \exp(f(j) + z_j), & \text{if } j \leq J_r \\ w y_R(z_{J_r}) & \text{if } j > J_r \end{cases} \tag{1}$$

$z_j = \rho z_{j-1} + \varepsilon_j$ with $\varepsilon_j \sim i.i.d.\ \ N(0, \sigma_\varepsilon^2)$, τ is the tax rate that funds the social security benefits, and $y_R(z_{J_r})$ is a function that approximates the US retirement system, as in Guvenen and Smith (2014). We assume that average hours per worker is chosen by the firms and workers are assumed to supply hours at no cost. As we specify in Section 2.2, w depends on the hours per worker chosen by the firm.

Households rent capital to the final good-producing firm and trade shares of the housing rental company. As we elaborate later, in perfect foresight equilibrium, they should yield the same rate of return, denoted as r_k .

to focus on a transition following two unexpected shocks, which is not easy but at least feasible to solve.

Housing Choices: Households enter the economy as *active* renters and can stay as renters by renting a house at the desired size at the price p_r per unit.[9] They can also purchase a house and become homeowners at any time. There is no unsecured borrowing in the model. However, households have access to the mortgage market to finance their housing purchases subject to a minimum down payment requirement. The terms of mortgage contracts, down payment, and mortgage pricing are endogenous and depend on household characteristics.

Homeowners can choose to stay as homeowners or become renters again, by either selling their houses or defaulting on mortgages. Homeowners can pay the existing mortgage or obtain a new one through refinancing, and can upgrade or downgrade their houses by selling the current house and buying a new one. Defaulting on a mortgage is possible but costly. After default, households become *inactive* renters; that is, they temporarily lose access to the housing market. Inactive renters become active renters with probability π. Therefore, agents have three statuses regarding their housing decision: homeowner, active renter, or inactive renter.

Several transaction costs are associated with housing market transactions. A seller has to pay φ_s fraction of the selling price. Obtaining a mortgage from banks involves a fixed cost (φ_f) and a variable cost (φ_m) as a fraction of the mortgage debt at the origination.

Fixed-rate Mortgages: For tractability, we assume that mortgages are due by the end of life, so that the household's age captures the maturity of the mortgage contract. We also allow for only fixed rate mortgages. Therefore, the mortgage contract can be characterized by its maturity and the periodic mortgage payment m. We assume that the mortgage payments follow the standard amortization formula computed at the bank lending rate r_ℓ. Thus, the relation between mortgage debt d and mortgage payment m in a period is given as

$$d = m\left(1 + \frac{1}{1+r_\ell} + \frac{1}{(1+r_\ell)^2} + ... + \frac{1}{(1+r_\ell)^{J-j}}\right) \Leftrightarrow m = d\frac{r_\ell\,(1+r_\ell)^{J-j}}{(1+r_\ell)^{J-j+1}-1} \qquad (2)$$

The remaining mortgage debt in the following period will be $(d-m)(1+r_\ell)$.

[9] The household sector builds on the ones in Arslan et al. (2015) and Guler (2015) but is extended in important ways, such as refinancing, flexible housing, and rental sizes..

The mortgage interest rate differs across households since households are heterogeneous. Ideally, the amortization schedule should be computed at the individual mortgage interest rate instead of r_ℓ. However, to avoid using an additional state variable, we assume that mortgage amortization is computed at r_ℓ, as in Hatchondo et al. (2015) and Kaplan et al. (2020). Then, individual default risk will show up in the pricing of the mortgages at the origination rather than in the mortgage interest rate. Thus, essentially all households pay a premium at the origination to reduce the mortgage interest rate to r_ℓ.

Optimization Problem of Households: We present the optimization problem of a purchaser here (the rest of the optimization problems are in Appendix C). If an active renter chooses to purchase a house, she chooses the mortgage debt level d that determines $q^m(d; a', h, z, j)$, the price of the mortgage at the origination, which is a function of the current state of the household (income realization z and age j), house size h, and asset choice a'. Then, the optimization problem of an active renter who chooses to buy a house is given by

$$V_j^{rh}(a, z) = \max_{c,d,h,a' \geq 0} \left\{ u(c, h) + \beta E V_{j+1}^h(a', h, d, z') \right\} \tag{3}$$

subject to

$$\begin{aligned} c + (1 + \delta_h)\, p_h h + \varphi_f I\,(d > 0) + \frac{a'}{1 + r_k} &= y(j, z) + a + d\,(q^m(d; a', h, z, j) - \varphi_m)\,, \\ d &\leq (1 - \iota)\, p_h h, \end{aligned}$$

where V^h is the continuation value for a homeowner, p_h is the house price, δ_h is the proportional maintenance cost of housing, ι is the down payment requirement, and I is the indicator function that takes a value of 1 if the mortgage debt is positive and a value of 0 otherwise.

2.2 Firms

A continuum of perfectly competitive firms produce output Y_t by combining capital K_t and labor N_t. The firm also chooses hours per worker (or worker utilization rate), u_t. The aggregate component of labor income $w(\overline{w}_t, u_t)$ (same as w in $y(j, z_j)$) is assumed to depend on the hours per worker, that is,

$$w(\overline{w}_t, u_t) = \bar{w}_t + \vartheta \frac{u_t^{1+\psi}}{1+\psi},$$

where ϑ and ψ are constants. In this formulation, hours are chosen by the firm, and workers are assumed to supply hours at no cost, but u_t, $\overline{w}_t$, and hence $w(\overline{w}_t, u_t)$ are determined in equilibrium.

Following Christiano and Eichenbaum (1992), Cooley and Quadrini (1999, 2004), Neumeyer and Perri (2005), Mendoza (2010), and Jermann and Quadrini (2012), we assume that the firm finances a fraction μ of the wage payment in advance from banks and pays interest on that portion. Then, the firm's problem is given by

$$\max_{K_t, N_t, u_t} \mathbb{Z}_t K_t^{\alpha} (N_t u_t)^{1-\alpha} - (r_{k,t} + \delta_k) K_t - (1 + \mu r_{\ell,t+1}) w(\overline{w}_t, u_t) N_t,$$

where $\mathbb{Z}_t$ is TFP, $r_{k,t}$ is the rate of return, and δ_k is the depreciation rate of capital.

The working capital requirement makes the firm's labor demand and production decision dependent on the bank lending rate. Since a worker's labor income depends on hours worked, labor income and output decline when the firm reduces work hours in response to an increase in bank lending rate r_ℓ.[10] We discuss the empirical support for this mechanism in Section 4.1.2.

We would like to conclude this section with two remarks. First, $w(\overline{w}_t, u_t)$ should not be considered as the wage rate per efficiency units of labor since changes in $w(\overline{w}_t, u_t)$ are driven by the work hours chosen by the firm. Instead, $w(\overline{w}_t, u_t)/u_t$ is the wage rate in the model. Second, even if changes in u_t can be modeled as a change in unemployment risk, almost all unemployment durations are short-lived. Since we will assume that each model period is two years, u_t captures the hours worked per worker during periods of both employment and unemployment in a two-year window.

[10] We could have achieved the same effect with endogenous labor supply. In that case, the firm would reduce labor demand, which would reduce wages. Households would reduce labor supply and output would decline. We choose this formulation because it is easier to handle computationally.

2.3 Rental Companies

A rental company enters period t with $(1-\delta_h) H_{t-1}^r$ units of rental housing stock where δ_h is the depreciation rate of housing. Then, it chooses H_t^r. In that period, the company receives net rent $(p_t^r - \kappa) H_t^r$ and pays dividend

$$x_t^r = p_t^h (1-\delta) H_{t-1}^r - p_t^h H_t^r - \frac{\eta}{2} p_t^h \left(H_t^r - H_{t-1}^r\right)^2 + (p_t^r - \kappa) H_t^r$$

to shareholders where p_t^r is the rental price per unit of housing and κ is the maintenance cost. The expression $\frac{\eta}{2} p_t^h \left(H_t^r - H_{t-1}^r\right)^2$ is the quadratic adjustment cost of changing rental supply. A higher η implies a more segmented housing market.

Since both capital and rental company shares are riskless in the steady state and along the transition path, both assets pay the same return except for the two unanticipated shock periods.[11] Given this, the first-order condition of the rental company gives the rental price as a function of house price and rental housing stock in periods $t-1$, t, and $t+1$:

$$p_t^r = \kappa + p_t^h + \eta p_t^h \left(H_t^r - H_{t-1}^r\right) - \frac{(1-\delta_h) p_{t+1}^h + \eta p_{t+1}^h \left(H_{t+1}^r - H_t^r\right)}{1 + r_{k,t+1}}. \quad (4)$$

This is the equation for the rental housing supply. The demand for rental housing comes from households' housing choices.

2.4 Banks

We assume that bankers are separate economic agents. The banking industry consists of a competitive continuum of identical banks, all of which are risk-averse and aim to maximize

$$\sum_{t=0}^{\infty} \beta_L^{t-1} \log\left(c_t^B\right),$$

[11] At the time of an unexpected shock, capital and the rental housing return could be different. Then, the realized return will be different from the contracted return, and these profits/losses are borne by the households proportional to their asset holdings.

where c_t^B is the banker's consumption. The risk aversion assumption leads to an interior solution for the banks' dynamic savings problem, enabling us to derive an analytical solution for their rich portfolio and savings choices.[12] There is no entry or exit from the banking sector. Banks finance their operations using their net worth ω_t and by borrowing B_{t+1} from the international market at a risk-free interest rate r_{t+1}. They provide loans L_{t+1}^f to firms at $r_{\ell,t+1}$, as well as issue mortgages and purchase existing mortgages.

Let $\theta = (d; a, h, z, j)$ define the type of a mortgage, ω_t be the bank's net worth, and $\ell_{t+1}(\theta)$ be the amount of investment in mortgage type θ (which includes any newly issued as well as existing mortgages). The budget constraint of the bank is given by

$$c_t^B + L_{t+1}^f + \int_\theta p_t(\theta)\,\ell_{t+1}(\theta) = \omega_t + B_{t+1},$$

where $p_t(\theta)$ is the price of a type-θ mortgage after the mortgage payment $m_t(\theta)$. The bank's net worth evolves according to

$$\omega_{t+1} = \int_\theta \int_{\theta'} v_{t+1}^l(\theta')\,\Pi(\theta'|\theta)\,\ell_{t+1}(\theta) + L_{t+1}^f(1 + r_{\ell,t+1}) - B_{t+1}(1 + r_{t+1}),$$

where

$$v_{t+1}^l(\theta') = m_{t+1}(\theta') + p_{t+1}(\theta')$$

and $\Pi(\theta'|\theta)$ is the endogenous transition probability governed by exogenous household characteristics and choices.

Banks can default at the beginning of a period by stealing a fraction ξ of their assets and not paying back their creditors. When it does so, it is excluded from banking operations in the future but can save at rate r_t. We denote the bank's value of default by $\tilde{\Psi}_{t+1}^D\left(\xi L'_{t+1}\right)$, where

$$L'_{t+1} = \left(\int_\theta \int_{\theta'} v_{t+1}^l(\theta')\,\Pi(\theta'|\theta)\,\ell_{t+1}(\theta) + L_{t+1}^f(1 + r_{\ell,t+1})\right).$$

[12]The ownership structure in our model greatly simplifies the analysis. Alternative ownership structures would introduce their own issues since we have a heterogenous-agent household economy. For example, we could assume that each household owns a share of the banking sector. However, since there is no representative household in our model, unlike papers that model banks as being owned by households, it is not clear which household's stochastic discount factor should be used for the banks' choice problems.

The expression $L_{t+1} = L^f_{t+1} + \int_\theta p_t(\theta)\, \ell_{t+1}(\theta)$ is the investment in t, and L'_{t+1} is the value of that investment in period $t+1$ after returns are realized. Investors lend to the bank up to a point where the bank does not default in equilibrium. Denoting the value to the bank of honoring its obligations by $\Psi_{t+1}(L_{t+1}, B_{t+1})$ and hence being able to continue its banking operations, the enforcement constraint is then given as[13]

$$\Psi_{t+1}(L_{t+1}, B_{t+1}) \geq \tilde{\Psi}^D_{t+1}\left(\xi L'_{t+1}\right).$$

The bank does not face any uncertainty in its net worth in the steady state or in a perfect foresight transition path even though each mortgage is risky because we assume a continuum within each household type, which translates into a continuum within each mortgage type θ.[14] Thus, an important property of the bank's problem is that all assets have to generate the same rate of return $r_{\ell,t+1}$. Therefore,

$$p_t(\theta) = \frac{1}{1 + r_{\ell,t+1}} \int_{\theta'} v^l_{t+1}(\theta')\, \Pi(\theta'|\theta) \text{ for all } \theta;$$

that is, the price of each mortgage is the expected present discounted value of its mortgage payments. Since the bank is indifferent between investing in any asset, we do not have to keep track of its asset distribution in the bank's problem. Then, we can show that $L'_{t+1} = (1 + r_{\ell,t+1})\, L_{t+1}$ and that the bank's enforcement constraint becomes

$$(1 - \phi_{t+1})(1 + r_{\ell,t+1})\, L_{t+1} \geq (1 + r_{t+1})\, B_{t+1},$$

which puts an endogenous upper bound on bank leverage.[15] This leverage constraint states that the bank can borrow up to a fraction of its assets and ϕ_{t+1} reflects the *haircut* on its

[13] The explicit expressions of $\Psi_{t+1}(L_{t+1}, B_{t+1})$ and $\tilde{\Psi}^D_{t+1}\left(\xi L'_{t+1}\right)$ are derived in Appendix F.

[14] Even if a bank invests in a θ-type households' mortgage by a tiny amount, its return is deterministic since a known fraction of θ-type households default. The continuum assumption grants us tractability while keeping the rich heterogeneity in the household sector.

[15] In Appendix F, we provide the characterization of the bank's problem in detail.

collateral, which increases with ξ.[16] The term ϕ_t is defined recursively as follows:

$$\phi_t = \xi^{1-\beta_L} \left((1 + r_{t+1}) \, / \, (1 + r_{\ell,t+1}) - (1 - \phi_{t+1}) \right)^{\beta_L} .$$

Then, the solution to the bank's problem is given as

$$L_{t+1} = \beta_L \widehat{\lambda}_t \omega_t \text{ and } B_{t+1} = \beta_L (\widehat{\lambda}_t - 1) \omega_t,$$

where

$$\widehat{\lambda}_t = \frac{(1 + r_{t+1})}{1 + r_{t+1} - (1 - \phi_{t+1})(1 + r_{\ell,t+1})}. \tag{5}$$

Perfect competition among banks implies that the present value of mortgage payments should be equal to the loan amount

$$dq^m(d; a', h, z, j) = \frac{1}{1 + r_{\ell,t+1}} \int_{\theta'} v^l_{t+1} (\theta') \, \Pi \, (\theta' | \theta)$$

at the time of the mortgage initiation. Given d and m, this equation solves for $q^m(d; a', h, z, j)$.[17]

We would like to conclude this section by a remark. Banks in our model do not face idiosyncratic and or aggregate risk. As a result, they do not hold extra buffers. With aggregate (and/or idiosyncratic) uncertainty, similar to Elenev et al. (2016), banks would choose to keep buffers to avoid hitting their constraints. However, the solution of the model with a rich heterogeneous household sector and aggregate or idiosyncratic bank risk would be overly complicated.[18]

[16] If the bank was not able to steal (i.e., $\xi = 0$), then $\phi_t = 0$ and the *collateral premium* (or, equivalently, the credit spread) $r_{\ell,t+1} - r_{t+1}$ would be zero.

[17] See Section F in Appendix for full derivation.

[18] With idiosyncratic bank risk (this is true with aggregate risk as well), the issue is that banks with different net worth (in a cross-section or over time) would price each type of household mortgage differently depending on their state. It is not clear how to solve such an equilibrium without making ad hoc assumptions on mortgage pricing and market structure.

2.4.1 Symmetric Equilibrium

We focus on a symmetric equilibrium in which each bank holds the market portfolio of mortgages. Since each bank's optimal consumption and investment choices are linear in its net worth, we obtain aggregation and can focus on the representative bank. In equilibrium, all economic agents maximize their objectives given bank funding cost r_{t+1} (assumed to be constant at r) and endogenous price sequences $\left\{r_{\ell,t}, r_{k,t}, \bar{w}_t, p_t^h, p_t^r\right\}_{t=1}^{\infty}$. The equilibrium market clearing conditions are given as follows.

Labor market: The labor market clears in all periods (i.e., $N_t = 1$).

Credit market: Let $\Gamma_t(\theta)$ be the distribution of available mortgages after households make their decisions at time t. In equilibrium, (i) $\ell_{t+1}(\theta) = \Gamma_t(\theta)$ (the representative bank holds the equilibrium mortgage portfolio), (ii) $L_{t+1} = \mu w(\bar{w}_t, u_t) + \int_\theta p_t(\theta)\, \Gamma_t(\theta)$, which determines $r_{\ell,t+1}$, and (iii) $A_{t+1} = K_{t+1} + V_{t+1}^{rc}(H_t^r)$, which determines $r_{k,t+1}$.

Housing market: Remember that total housing supply is fixed at H. Thus, the total demand of owners and renters should be equal to the supply, which determines the house price $p_h(t)$. Given the house price $p_h(t)$ and the rental price $p_r(t)$, households solve their optimal housing choices, which gives the demand for owner-occupied units $H_t^{o,\mathrm{D}}$ and rental units $H_t^{r,\mathrm{D}}$. The supply of rental housing units is given by the first-order condition of the rental company (equation 4). Then, the following two equilibrium conditions give the house price p_t^h and rental prices p_t^r:

$$H_t^{r,\mathrm{S}} = H_t^{r,\mathrm{D}} \text{ and } \bar{H} = H_t^{r,\mathrm{D}} + H_t^{o,\mathrm{D}}.$$

Government: The government runs a pay-as-you-go pension system. It collects social security taxes from working-age households and distributes to retirees. We assume that the pension system runs a balanced budget:

$$\sum_{j=1}^{J_R}\sum_{z}\tau y\left(j,z\right)\pi_j\left(z\right)=\sum_{j=J_R+1}^{J}\sum_{z}y_R\left(j,z\right)\pi_j\left(z\right),$$

where $\pi_j\left(z\right)$ is the measure of individuals with income shock z at age j.

Finally, note that a bank is a leveraged investor. Banks borrow the amount $\widehat{\lambda}_t-1$ per unit of their net worth and earn an excess return $r_{\ell,t+1}-r$ on this amount in addition to $r_{\ell,t+1}$ they earn from their own net worth. Thus, a banker's gross return on net worth at time t is equal to $1+r_{\ell,t+1}+(\widehat{\lambda}_t-1)(r_{\ell,t+1}-r)$. In the steady state, we have $1+r_\ell+(\widehat{\lambda}-1)(r_\ell-r)=1/\beta_L$, which we use with the excess return $r_\ell-r$ and the leverage rate $\widehat{\lambda}-1$ from the data to pin down the banker's discount rate in the calibration section.

3 Calibration

A model period is two years. Households start the economy at age 25, work until they retire at age 65, and live until age 85. Table 1 presents externally set and internally calibrated parameters under the columns labeled "External" and "Internal" respectively.

Preferences: We assume that households receive utility from consumption and housing services captured by the following CES utility specification: $u\left(c,s\right)=\left((1-\gamma)c^{1-\epsilon}+\gamma s^{1-\epsilon}\right)^{\frac{1-\sigma}{1-\epsilon}}/(1-\sigma)$. We choose $\epsilon=1$, which implies a unit elasticity of substitution between housing and consumption, consistent with the estimates in Piazzesi et al. (2007). Following the literature, we set $\sigma=2$, which implies an elasticity of intertemporal substitution of 0.5.[19] We calibrate γ to match the share of housing services in GDP as 15 percent and the discount factor β to match the capital-output ratio of 1 in our biennial model.

Income Process: For the income process before retirement, we set the income process parameters $\rho=0.92$ and $\sigma_\varepsilon=0.31$, which correspond to an annual persistence of 0.96

[19] In Figure 8 in Appendix B.2, we report the dynamics of house prices, output, and consumption for different values of σ and ϵ.

Table 1: Parameters (externally set and internally calibrated)

		Value	
Parameter	Explanation	External	Internal
σ	risk aversion	2	
α	capital share	0.3	
ψ	curvature on hours	0.5	
ρ_ε	persistence of income	0.92	
σ_ε	std of innovation to AR(1)	0.31	
φ_h	selling cost for a household	7%	
φ_e	selling cost for foreclosures	25%	
ζ	fixed cost of mortgage origination	2	
δ_h	housing depreciation rate	5%	
τ	variable cost of mortgage origination	0.75	
η	rental adjustment cost	3	
π	prob. of being an active renter	0.265	
$\bar{H}$	housing supply	1.0	
ι	down payment requirement	0	
β	discount factor		0.88
$\underline{h}$	minimum house size		0.78
r	deposit rate		6.52%
γ	weight of housing services in utility		0.19
μ	share of wage bill financed from banks		0.81
β_L	bank discount factor		0.73
ξ	bank seizure rate		0.25
κ	rental maintenance cost		0.04
δ_k	capital depreciation rate		0.20

Table 2: Moments

Statistic	Data	Model
Capital-output ratio	1	1
Homeownership rate–aggregate	64%	64%
Mortgage debt to GDP ratio	40%	40%
House price to GDP ratio	0.825	0.825
Share of housing services in GDP	15%	15%
Ratio of mortgage loans to total loans in bank assets	0.45	0.45
Mortgage premium	0.03	0.03
Bank leverage ratio	10	10
House price-rental price ratio	5.5	5.5
Non-residential investment-output ratio	20%	20%

and a standard deviation of 0.17 following Storesletten et al. (2004). Retirement income approximates the US retirement system, as in Guvenen and Smith (2014).

Production Sector: We set the capital share in output to $\alpha = 0.3$. We target a capital-output ratio $\frac{K}{Y}$ as one, which corresponds to a capital-output ratio of 2 annually. We normalize $N = 1$, and $\mathbb{Z} = 1$ and target $u = 1$ at the steady state. Then, since $Y = \mathbb{Z}K^{\alpha}(Nu)^{1-\alpha}$, we get $Y = K = 1$.

The share of housing services in GDP is 0.15. Since in our model, GDP, which includes the imputed income from housing, corresponds to $Y_A = Y + p_r\bar{H}$, this results in $Y_A = \frac{1}{0.85}$ and $p_r\bar{H} = \frac{0.15}{0.85}$. In the data, the ratio of non-residential investment to GDP is 0.2. Since, at the steady state, this ratio is $\frac{\delta_k K}{Y}$, this gives us a capital depreciation rate of $\delta_k = 0.2$ biennially. Given these targets, the model-implied biennial return to capital becomes $r_k = \alpha\frac{Y}{K} - \delta_k = 10$ percent.

We calibrate the labor utilization function curvature ψ to match the response of hours in the model to the data. We choose $\psi = 0.5$ with which the model generates an employment decline of 1.8 percent in response to a 1 percentage point increase in the bank lending rate, which falls in the middle of the employment effect found in Gertler and Gilchrist (2018).[20] We target $u = 1$ in the steady state. From the firm's problem, $\vartheta = \left(\frac{1-\alpha}{1+\mu r_\ell}\right)\left(\frac{\alpha}{r_k+\delta}\right)^{\frac{\alpha}{(1-\alpha)}}$ gives

[20]See Section B.2 in Appendix for robustness with respect to ψ and several other parameters.

the calibrated value of ϑ.

Housing Market: The probability of an inactive renter becoming an active renter is set to 0.265 to capture the fact that the bad credit flag remains for about seven years in the credit history of households. We set the selling cost (φ_s) to 7 percent for regular sales and to 25 percent for foreclosed properties, consistent with the estimates of Campbell et al. (2011). We set the fixed mortgage origination cost $\zeta = 2$ percent of GDP and the variable cost of mortgage origination $\tau = 0.75$ percent of the mortgage loan (Federal Reserve Board (2008)). We set the down payment requirement to zero since there is no explicit regulation for down payment. However, in the model many households choose to make some down payment in order to get favorable mortgage terms.

The ratio of house prices to biennial rental payments is set to 5.5 (Sommer et al. (2013)). This moment, together with the fact that the ratio of housing services to GDP is 0.15, implies $p_h\bar{H} = \frac{p_r\bar{H}}{Y_A} \times Y_A \times \frac{p_h}{p_r} = \frac{0.15\times5.5}{0.85}$=0.97. We set $\bar{H} = 1$, which gives us $p_h = 0.97$. Then, we calibrate γ to clear the housing market.[21] We set the biennial depreciation rate for housing units as $\delta_h = 5$ percent (Harding et al. (2007)). The steady-state relation between the rental price and house price is given by $p_r = \kappa + \frac{r_k+\delta_h}{1+r_k}p_h$. This gives us an estimate of κ given our target $\frac{p_h}{p_r} = 5.5$. We calibrate the minimum house size for owner-occupied units to be $\underline{h}$ to match a homeownership rate of 64 percent.

As we stated earlier, η captures the degree of housing market segmentation since it is the cost of changing rental housing supply (by converting owner-occupied housing to rental housing and vice versa). In a recent paper, Greenwald and Guren (2020) document empirical evidence that housing market is close to being fully segmented. We have experimented with different values of $\eta \in \{1, 3, 5\}$. It turns out that the dynamics of output, house prices, and consumption do not change significantly across different values of η (see Section B.2 in Appendix for robustness). The model generates larger declines in price-rent ratio in the bust

[21]This is the same as calibrating γ to match the ratio of housing services to GDP to be 0.15. We use the same trick to calibrate the discount factor. Instead of targeting the capital output ratio, we impose it in the calibration, which gives us the equilibrium capital rental rate, and calibrate the discount factor to clear the asset market. This method allows us to pin down the equilibrium prices directly in the calibration stage without the need of an additional iteration to find the equilibrium prices for each set of parameter configuration.

with higher values of η since rents move less than house prices, which is more consistent with the data. Nevertheless, the differences between $\eta = 3$ and $\eta = 5$ are small. Thus, we choose $\eta = 3$ as our benchmark parameter.

Financial Sector: Since not only banks but also other institutions hold large amounts of mortgage-related products, we follow Shin (2009) and include deposit-taking institutions (US chartered depository institutions and credit unions), issuers of asset backed securities, GSEs, and GSE-backed pools from FED Z1 data in our bank definition. Then we match bank balance sheets to the 1985-1994 average in the data. We use Tables L.218 and L.219 to obtain the total amount of home and multifamily residential mortgages held by banks. Banks on average hold \$2.117 trillion of these mortgages, which correspond to 86 percent of all mortgages (stable from 1985 to 1994). To compute the amount of lending to non-financial firms, we use the balance sheets of non-financial firms (Table L.102). We use total loans (loans from depository institutions, mortgages, and other loans), which average to \$2.245 trillion, and miscellaneous liabilities, which average to \$1.23 trillion. Residential mortgages constitute 49 percent of banks' balance sheets if we include the loans only and 39 percent if we also include miscellaneous liabilities as firms financing from banks. Thus, we choose $\frac{\int_\theta p_t(\theta)\Gamma_t(\theta)}{\mu w(\bar{w}_t,u_t)N_t+\int_\theta p_t(\theta)\Gamma_t(\theta)}$ (the ratio of mortgages to banks' total financial assets) as 45 percent, which gives μ, the fraction of wage bill financed through banks.

In the steady state, we have $r_\ell - r = \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$, where $\widehat{\lambda} = \frac{(1+r)}{1+r-(1-\phi)(1+r_\ell)}$ is the endogenous leverage ratio and $\phi = \xi^{1-\beta_L}\left({(1+r)}/{(1+r_\ell)} - (1-\phi)\right)^{\beta_L}$ is the haircut. We calibrate r to match a mortgage debt toGDP ratio of 40 percent (corresponding to an 80 percent ratio annually), and we target $r_\ell - r = 3$ percent, representing the average biennial gap between the 30-year mortgage interest rate and the 10-year Treasury rate in the data. We also target the bank leverage ratio $\widehat{\lambda}$ as 10 following Gertler and Kiyotaki (2015). These two targets give us the bank's discount factor β_L and the bank's seizure rate ξ.[22]

Overall, we have 9 parameters that we calibrate internally: discount factor (β), minimum

[22] Gertler and Kiyotaki (2015) and many other papers introduce bank survival risk to their models effectively reducing banks' discount factor. Since we have not incorporated the survival risk, calibrated bank's β_L is smaller.

house size ($\underline{h}$), deposit rate (r), weight of housing services in utility (γ), share of wage bill financed by banks (μ), bank's discount factor (β_L), bank's asset seizure rate (ξ), maintenance cost for rental units (κ), and capital depreciation rate (δ_k). The last four of these parameters are identified directly through analytical moments obtained through the model as discussed above. This leaves us with five parameters that we calibrate using the model simulated data to jointly match the following five data moments (Table 2): 64 percent average homeownership rate, 40 percent mortgage debt to GDP ratio, capital-output ratio of 1, share of mortgages in bank balance sheet as 45 percent, and share of housing services in GDP as 15 percent.

Leverage Shock: The boom and bust periods coincided with important changes in financial markets that shifted credit supply.[23] The Glass-Steagall Act, the bill that separated banking activities from investment banking ones, was repealed in 1999 after being loosened for about a decade. As a result, deposit-taking banks had the opportunity to extend their balance sheets. On the securitization side, from 1995 to 2005, the volume of private-label mortgage-backed securities increased dramatically from negligible levels to $1.2 trillion but disappeared with the crisis. The mortgage-backed securities were considered to be very safe during the boom leading to the crisis, which allowed financial institutions to use them as collateral and expand leverage. We view both the regulatory changes and changes in investor sentiment toward mortgage-backed securities as the driving force behind the expansion and then contraction of funding to the banking system. Nevertheless, the distinction between the two is not critical. In the model, ξ directly influences the bank leverage, which is observed in the data.[24] As a result, we calibrate the changes in ξ to match the changes in bank leverage without taking a stand what the underlying reason is.

To study the role of bank credit supply, we assume that the economy is at the steady state before 1998, but in 1998, unexpectedly, bank leverage starts linearly increasing for T years and is expected to remain constant after that. Unexpectedly, in 2008, however, the

[23]See Chernenko et al. (2014) for developments in the securitization market and Sherman (2009) for important changes in financial market regulation in the US.

[24]A lower (higher) value for ξ allows banks to have a higher (lower) leverage.

leverage reverts back. We calibrate the changes in parameter ξ and T to match the increase in the financial system book leverage and the decline in the bank credit spread $(r_\ell - r)$ from 1998 to 2006.

We refer to two sources for the financial system book leverage. First, Federal Reserve Bank of New York (2020) documents that the leverage ratio of the consolidated US banking organizations has increased by 25 percent from the first quarter of 1996 to the last quarter of 2007. We use the leverage ratio of all institutions (see page 34 of the report). Second, the Financial Stability Report by Federal Reserve Board (2019) documents that the leverage ratio of security brokers and dealers has increased by 50 percent from the first quarter of 1995 to the first quarter of 2008 (see Figure 3-5 in the report).

Both studies report marked-to-book leverage. However, in our model bank assets, L_{t+1}, and net worth, ω_t, are in market values and the ratio L_{t+1}/ω_t gives the marked-to-market leverage, which is the same as the book leverage when the economy is in steady state.[25] We calibrate the changes in parameter ξ to have an increase in the financial system book leverage for 37.5 percent (from 1996 to 2006), which falls in the mid-range of 25 percent and 50 percent. The top left panel in Figure 2 compares the book leverage from our model and these sources.[26]

The credit spread in our model is comparable with the EBP developed in Gilchrist and Zakrajšek (2012a), which displays an overall decline of 0.5 percentage points in 1990s until 2007, excluding the spike that happened in the Dotcom bubble burst. Thus, we target a decline in credit spread of 0.5 percentage points during the boom period.[27]

[25] However, after unexpected shocks, market and book values will no longer be equal. To be able to compare the model to the data, we compute the book values of bank loans and net worth and calculate the corresponding book leverage in our model.

[26] In our framework, the leverage constraint and haircuts on collateralized loans are tightly linked. Available data suggest that haircuts more than doubled for most mortgage-related securities after the crisis (Committee on the Global Financial System (2010)), consistent with the leverage dynamics in our model.

[27] The mortgage interest rates fell by one percentage point from 1998 to 2006 in the data (Justiniano et al. (2017)). In principle, we could add a decline in the bank funding rate, another credit supply shock, so that the model generates a one-percentage-point decline in the bank lending rate. However, with this larger decline, the boom and the subsequent bust are more pronounced. Thus, our results provide a lower estimate for the importance of credit supply shocks for the boom-bust. We have chosen not to add a decline in the bank funding rate to generate a one-percentage-point decline in the mortgage rate since, with this larger boom-bust, the bank net worth becomes negative during the bust in the absence of government interventions. However, the decomposition exercises become more involved with government interventions without altering

We conclude this section with a remark: Shocks to house price expectations as in Kaplan et al. (2017) might have also contributed to the boom-bust. In principle, we can include both shocks and compute the fraction of the boom-bust accounted for by each shock. We have indeed conducted such an experiment, but it has a tiny effect on the importance of the credit supply shock (see Appendix B.3 for calibration and results of this experiment). Moreover, this two-shock experiment leads banking sector to collapse in the bust without government bailouts. However, as we just noted, the existence government bailouts complicates the analysis. Thus, we have chosen to use only the credit supply shock, studying its importance and amplification channels, which provides much cleaner decomposition results, and relegate the analysis with two shocks to Appendix B.3.

4 Benchmark Results

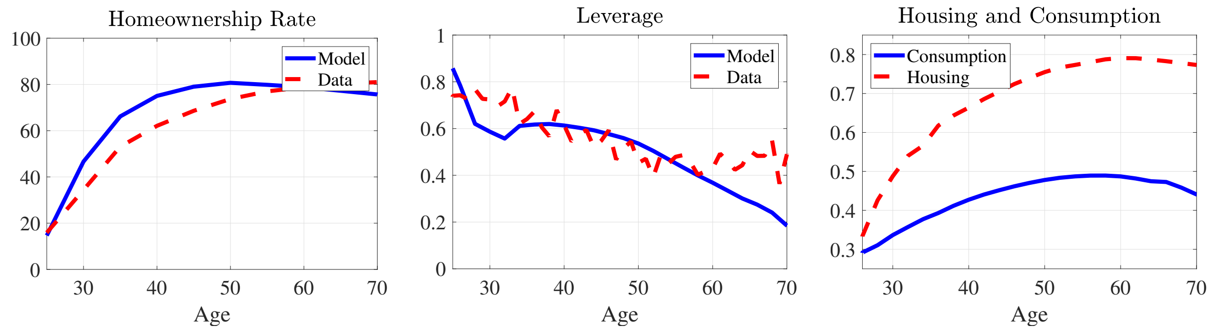
Before turning to the analysis of transition dynamics, it is useful to check the model's performance in matching some key life-cycle statistics that may be important for the soundness of the quantitative exercise.

Overall, the life-cycle implications of the model closely match the data (Figure 4). The homeownership rate increases over the life cycle, similar to the data. Household leverage, measured as the ratio of mortgage debt to house price, declines with age in both the data and the model. Average consumption and housing consumption in the model more than double over the life cycle and are very close to the values reported in the literature, such as in Aguiar and Hurst (2013).

Transmission of the Leverage Shock: The changes in the bank lending rate r_ℓ are the key mechanism through which the leverage shock transmits to the economy. A lower r_ℓ, for example, implies lower borrowing costs for both households and firms. Consequently, households' demand for consumption and housing increases, representing the *direct* effect of lower r_ℓ. Furthermore, as firms demand more labor, households' labor income increases.

our main conclusions because the effect of the government interventions essentially differences out.

Figure 1: Life-Cycle Properties of the Model



Notes: The graph shows the life-cycle properties of housing and mortgage debt. The left panel plots the homeownership rate. The middle panel plots mortgage debt relative to housing value. The data come from 1995 Survey of Consumer Finances. The right panel plots the consumption and housing expenditure at a given age.

Hence, households demand more consumption and housing, reflecting the indirect effect of lower r_ℓ.

Exogenous shocks to bank leverage translate into one-for-one changes in the bank lending rate r_ℓ in the absence of any feedback from bank balance sheets. Focusing first on the steady state, an increase in bank leverage decreases the credit spread according to $r_\ell - r = \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$. Thus, a permanent increase in $\widehat{\lambda}$ will lead the economy to a steady state with a permanently lower r_ℓ. Moreover, when the bank net worth effects are absent, the changes in $\widehat{\lambda}$ will translate into changes in r_ℓ during the transition, as given by this equation. As a result, r_ℓ gradually falls during the boom and is expected to stay low permanently.

The leverage parameter reverts unexpectedly and permanently to its steady state level at the time of the bust, which translates into a permanent increase in r_ℓ. However, the deterioration of bank balance sheets amplifies the increase in r_ℓ during the bust. We explain this amplification mechanism next.

While all variables affect each other simultaneously, it is instructive to proceed with an iterative approach to demonstrate the amplification mechanism during the bust. For this purpose, remember that the bank net worth in period t is given as

$$\omega_t = \int_\theta \int_{\theta'} \left(m_t\left(\theta'\right) + p_t\left(\theta'\right)\right) \Pi\left(\theta'|\theta\right) \Gamma_{t-1}\left(\theta\right) + L_t^k\left(1 + r_{\ell,t}\right) - B_t\left(1 + r_t\right).$$

The shock that generates the bust is a decrease in $\widehat{\lambda}_t$ back to its steady-state level, reducing loan supply through $L_{t+1} = \beta_L \hat{\lambda}_t \omega_t$.[28] As a result, the equilibrium bank lending rate $r_{\ell,t+1}$ increases. Since all assets should pay the same expected return in a perfect foresight equilibrium, which is the case during the transition after the unexpected shock, mortgage values decline according to $p_t(\theta) = \frac{1}{1+r_{\ell,t+1}} \int_{\theta'} (m_{t+1}(\theta') + p_{t+1}(\theta')) \Pi(\theta'|\theta)$ for all θ. Thus, a higher $r_{\ell,t+1}$ further reduces the bank's net worth at time t. In response, loan supply L_{t+1} declines further, and $r_{\ell,t+1}$ increases more. With higher $r_{\ell,t+1}$, mortgage valuations and bank net worth decline further, leading to additional increases in $r_{\ell,t+1}$. Furthermore, the increase in foreclosures additionally reduces bank net worth. This is the key mechanism through which the deterioration of bank balance sheets amplifies the transmission of a shock to bank leverage. However, the spike in r_ℓ and the sharp drop in bank net worth are short-lived because the amplification mechanism described above works in the opposite way in the recovery. When r_ℓ starts to decline, the market value of the bank's mortgage portfolio begins to recover. This increases the bank's net worth, hence credit supply, reducing r_ℓ's even more. As a result, bank net worth recovers quickly.

4.1 Boom-Bust Dynamics of Aggregate Variables

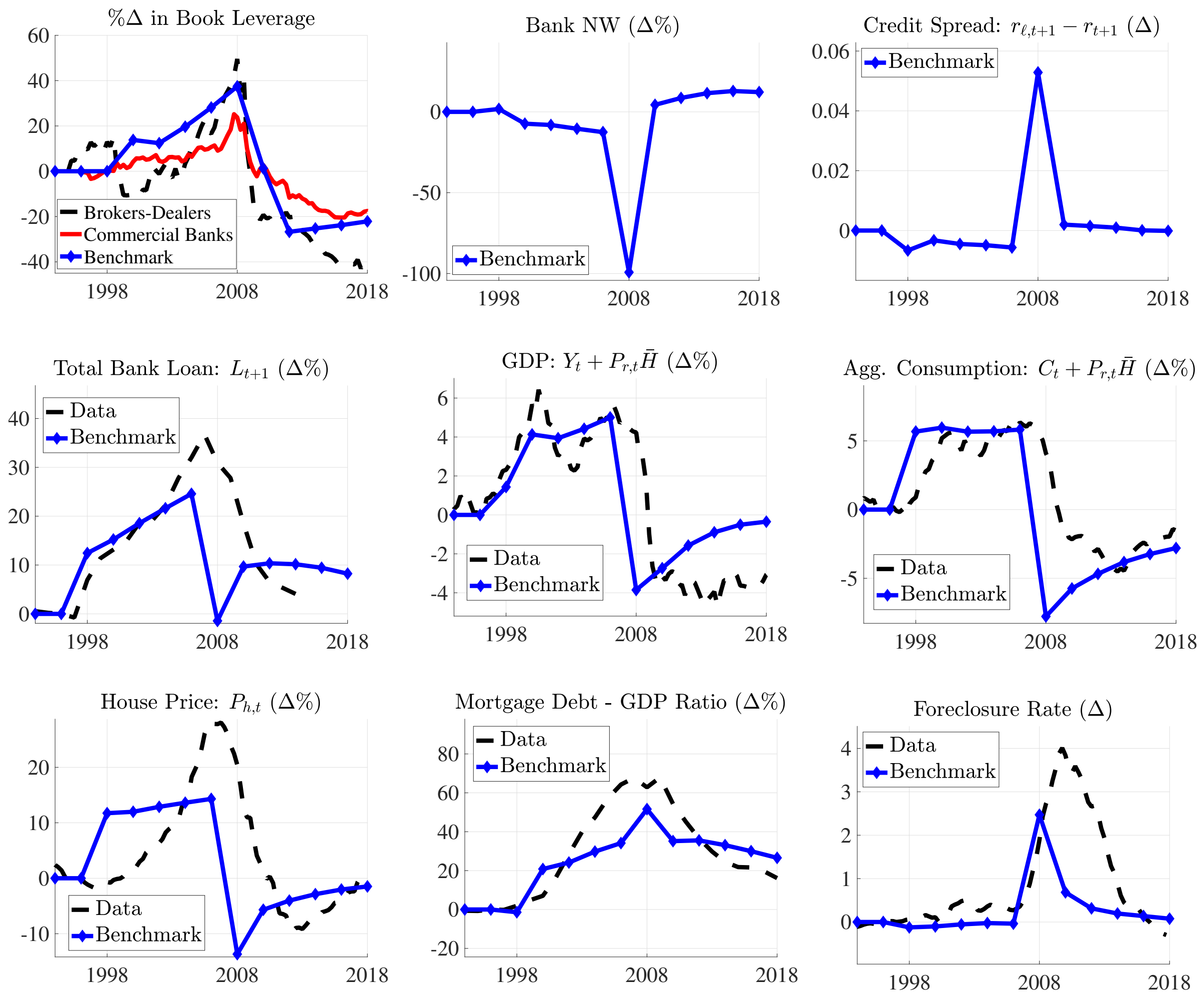
The benchmark economy experiences a significant boom-bust in the banking, housing, and real sectors in response to the credit supply shocks. We begin by discussing the dynamics of aggregate variables in these sectors, as presented in Figure 2.

4.1.1 Banking Sector Dynamics

The top-left panel in Figure 2 shows the bank book leverage, which is calibrated to increase by 37.5 percent. Even through not calibrated, the book leverage declines by 42.8 percent in the subsequent two periods and recovers slowly, consistent with the data. The leverages of commercial banks and security brokers and dealers in the data decline by 27 and 71 percent

[28] The term $\widehat{\lambda}_t$ is an endogenous object determined by equation (5). The parameter that goes back to its steady-state level is ξ, which eventually decreases $\widehat{\lambda}_t$ to its initial steady-state level.

Figure 2: Boom-Bust Dynamics



Notes: The graph plots the dynamics of key variables during the boom-bust episode. Credit spread is measured annually. Data counterparts of bank loans, output, consumption, and house price are percentage deviations from their linear trends obtained from the 1985-2006 period. Total data for bank loans include home and multifamily residential mortgages, and firm loans and miscellaneous liabilities. The data for the book leverage of banks are from Federal Reserve Bank of New York (2020); data for security brokers and dealers are from Federal Reserve Board (2019). In the text, we compute the bust changes relative to the peak of the boom.

from 2008 to 2010, respectively.[29]

With higher leverage, bank loan supply increases, and the bank lending rate r_ℓ declines. The bank net worth declines during the boom since the bank funding cost r has not changed and r_ℓ is lower. Thus, the banking sector supports more credit with lower bank net worth but with higher debt. The model generates a 25 percent rise in bank loans in the boom. The share of mortgages in banks' portfolios also increases.

The crisis occurs as the leverage constraint reverts to its initial steady-state level. The credit spread $r_\ell - r$ jumps by 5.9 percentage points and mortgage valuations decline, and bank net worth sinks almost to zero, with banks barely surviving the crisis period. This increase in the credit spread is higher than the 3 percentage point increase in the EBP documented in Gilchrist and Zakrajšek (2012a) and Gertler and Gilchrist (2018). However, it is important to note that during the 2008 Financial Crisis, there were several government interventions that prevented banks' net worth from declining as much as it does in our model. Including those interventions would complicate the analysis without altering the contribution of the credit supply shock. These interventions would essentially be differenced out, as we would be fixing them and computing the effect of the credit supply shocks.

Finally, since mortgages are long-term assets, banks cannot fully adjust their balance sheets by holding fewer mortgages. Therefore, they reduce their lending to firms by 36.3 percent. Overall, the model accounts for a substantial portion of the reduction in total bank loans to firms and households.

The dynamics of the bank lending rate are consistent with other empirical findings. During the boom, interest rates on firm loans and mortgages declined (Glaeser et al. (2012) and Justiniano et al. (2017)). Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, the lending interest rate on loans more than quadrupled (Adrian et al. (2013)).

[29] Consistent with what we report here, there is broad agreement that marked-to-book leverage is procyclical (Adrian and Shin (2010), Nuno and Thomas (2017), and Coimbra and Rey (2017)). On the other hand, the marked-to-market bank leverage (L_{t+1}/ω_t) spikes at the time of the bust as bank net worth sinks, which is also consistent with the findings of Begenau et al. (2018).

4.1.2 Real Sector Dynamics

At the peak of the boom, the US per capita GDP was almost 6 percent above its trend. GDP ($Y_A = Y + P_r\overline{H}$) in the model increases by 5.0 percent (Figure 2, middle row middle panel) during the boom (output Y increases by 5.2 percent). With the decline in borrowing rates, households' debt payments decline since they obtain new mortgages and can refinance at lower interest rates. That allows households to afford more consumption and increase savings. Hence, aggregate capital increases by 9.0 percent during the boom, which accounts for half of the increase in output Y. The remainder is accounted by the decline in bank lending rate. With the crisis, GDP falls by 8.4. The aggregate labor income follows a boom-bust pattern similar to the output, which is critical for the boom-bust in house prices and consumption, as we demonstrate in Section 4.2.1.

Aggregate consumption ($C_A = C + P_r\overline{H}$) increases by 5.8 percent during the boom and declines by 12.8 during the crises (Figure 2, middle row right panel).[30] The immediate response of consumption to the shocks is driven by the immediate response of house prices to new information from two unexpected shocks. Since house prices are one of the key determinants of consumption (together with labor income), their jump and fall with the boom and bust create a non-smooth consumption pattern.[31]

The response of the firms' labor demand to the changes in the bank lending rate is one of the key drivers of the boom-bust in the production sector. There is substantial evidence that the labor decisions of firms are indeed influenced by financial conditions, which corroborates the predictions of our model. For example, Chodorow-Reich (2013) finds that firms that worked with weaker banks prior to the crisis reduced employment more. Benmelech et al. (2019) find similar evidence from the Depression era, and Popov and Rocholl (2015) bring evidence from Germany during the 2008 crisis. Finally, Ivashina and Scharfstein (2010) document a more than 50 percent decline in bank capital expenditure and working capital

[30] Since the bank borrowing rate r is exogenous, this model is essentially an open economy where output is given as $Y_A = C_A + I + NX$. In 1998, NX declines and stays negative. Thus, the model implies that there is a net capital inflow to the US during the boom period, which is broadly consistent with the data.

[31] When we feed the observed decline in house prices as an exogenous shock while keeping all other prices constant in their boom sequence, the implied elasticity of consumption to house prices in our model turns out to be 0.23, which is consistent with the findings in Berger et al. (2018).

loans to corporations, and Adrian et al. (2013) find that capital expenditure and working capital loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis. At the macro level, Gertler and Gilchrist (2018) find that a shock to in the EBP generates a significant drop in employment in the subsequent 10 quarters. And, Gilchrist and Zakrajšek (2012b) find that an increase in the EBP leads to a drop in real GDP growth over the subsequent 4 quarters.

4.1.3 Housing Market Dynamics

House prices increase by 14.3 percent during the boom (Figure 2, bottom row left panel), accounting for half of the increase in house prices in the data. As a result of higher house prices, lower down payment ratios, and the jump in refinancing activity, the mortgage-debt-to-GDP ratio (Figure 2, bottom row middle panel) increases by 40.9 percent in the model.[32] The bust in house prices is much deeper: house prices decline by 24.5 percent from their peak to 14.3 percent below their initial steady-state level and slowly recover. During the bust, both debt and leverage gradually decline back to their initial steady-state levels. The foreclosure rate in the model stays low during the boom and jumps by 2.5 percentage points in the bust period, accounting for more than 60 percent of the increase in the data.

4.2 The Drivers of the Boom-Bust

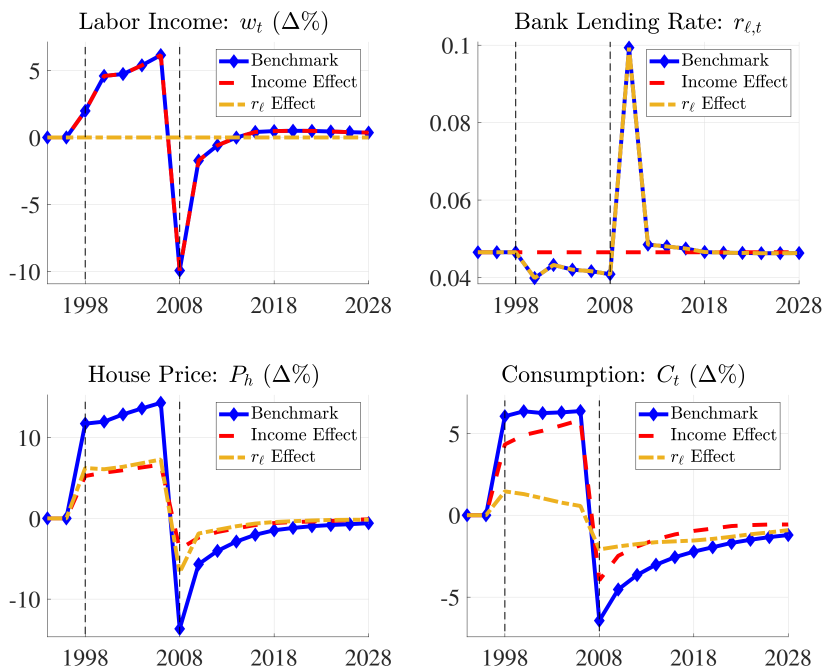
In this section, we explore two mechanisms that are relevant for the boom-bust in house prices and consumption: (i) equilibrium feedback from credit supply to household labor income and (ii) the amplifications arising from the deterioration of bank balance sheets during the bust.

4.2.1 The Roles of Labor Income and the Bank Lending Rate

The changes in bank leverage influence model dynamics through their effects on the bank lending rate. The changes in the bank lending rate, in turn, affect households both "directly"

[32] Household leverage increases by much less since house prices also increase.

Figure 3: The Role of Labor Income and Bank Lending Rate



Notes: The graph plots the dynamics of wages, house prices, and consumption. For the "Income Effect" exercise, we impose the same wage dynamics that arise in the benchmark economy and keep the lending rate, r_ℓ, at the steady-state level. For the "r_ℓ Effect" exercise, we keep labor income at the steady-state level, as shown in top left panel, and shock the economy with r_ℓ boom and bust sequences of the benchmark economy (top right panel).

via borrowing costs and "indirectly" through affecting labor income.[33] To isolate these direct and indirect effects, we solve two versions of our model where we keep the aggregate component of household labor income or the bank lending rate constant at their initial steady-state levels and analyze how the boom-bust cycles differ from our benchmark economy. We present the results of this analysis in Figure 3.

Our results suggest the indirect effect of the bank lending rate, through changes in labor income, account for about half of the size of the boom-bust in house prices, and most of the boom-bust in consumption in the benchmark economy. Its direct effect is equally important for house prices; however, its direct effect on consumption is limited: a 1 percent increase in consumption during the boom and a 2.5 percent decline during the bust. Thus, the most of the increase in household consumption is driven by the indirect effect.

[33]The 6.1 percent increase in labor income during the boom and the 15.2 percent decline during the bust, as firms adjust their labor demand in response to the changes in the cost of funding, affect households' consumption and housing demand.

Kaplan et al. (2020) study a similar framework and reach the conclusion that credit conditions (LTV, PTI, mortgage origination, and temporary interest rate shocks) cannot generate a significant boom-bust cycle in house prices. In fact, similar to them, we also find small of effects of these shocks on house prices. However, our analysis differs from theirs in three important aspects. First, the credit supply shock in our framework is not an isolated shock to the household borrowing rate; it also affects labor income. Our findings in this section suggest that about half of the effects on house prices are driven by this channel. The second major difference is the persistence of the shocks. Not surprisingly, the boom-bust cycle gets amplified when the shock is more persistent.[34] Finally, there is a critical difference between LTV and PTI shocks and permanent changes in the bank lending rate. Relaxation of LTV and PTI constraints shift housing demand from renting to owning. Since households can rent a house of the desired size, these shocks do not significantly affect aggregate housing demand.[35] On the other hand, a permanently lower bank lending rate creates a significant income effect—since mortgage payments decline for a given debt amount–and a wealth effect—since labor income permanently increases—, and thus, increases the total housing demand. Finally, the deterioration of bank balance sheets significantly amplify the bust in our model.

4.2.2 The Role of Bank Balance Sheet Deterioration in Amplifying the Bust

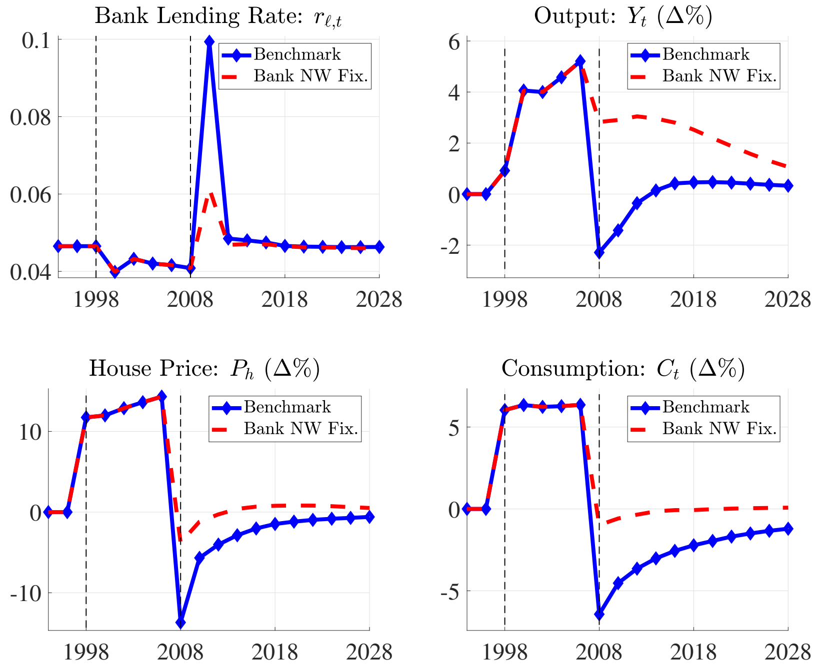
In the bust period, the credit supply declines not only because of the exogenous tightening of the bank leverage constraint but also because of the endogenous deterioration of bank balance sheets, which significantly amplifies the bust. In this section, to quantify the role of the deterioration of bank balance sheets on the aggregates during the bust, we eliminate the decline in bank net worth in the bust and analyze the equilibrium transition.[36] The red

[34] We have experimented with temporary shocks to bank leverage. The effects on housing market as well as on the rest of the economy are much smaller with temporary leverage shocks.

[35] Relaxing these constraints does not generate any boom in our model. To check whether tightening them generates a bust, we have conducted an experiment in which the LTV limit is reduced from 100 percent to 80 percent, and a PTI ratio of 25 percent is imposed (the benchmark does not have a PTI limit) with 0.6 persistence. Despite their large sizes, the LTV shock generates a small effect on aggregates, while the PTI shock has almost no effect (see Figure 7 in Appendix B.1).

[36] Essentially, we solve the transition of the economy starting with the bust distribution but with bank net worth fixed at the boom level in the first period of the bust. We focus on the bust period only, since the

Figure 4: The Effect of Bank Balance Sheet Amplification on the Bust



Notes: These graphs plot the contribution of bank balance sheets to some of the key variables of the model. For the benchmark economy (blue diamond lines), the shock is an unexpected ease and then an unexpected reversal of borrowing-lending constraints of the banks. The red lines are the model-implied dynamics where we fix the bank net worth in 2008 (bust) to the 2006 (boom) level. The difference between the two lines measures the amplification arising from bank balance sheet deterioration.

line in Figure 4 shows the model dynamics for this exercise, and the blue line shows the dynamics for the benchmark. The difference between the blue and red lines indicates the role of bank balance sheet deterioration in amplifying the bust, which we also report under the "Contribution of BBS" column in Table 3.

Overall, we find that the deterioration of the bank balance sheet significantly amplifies the bust. Specifically, we find that the bank balance sheet channel amplifies the drop in output by 61 percent because of the lower labor demand caused by the spike in the bank lending rate at the time of the bust. The bank balance sheet deterioration accounts for 35 percent of the 24.5 percent decline in house prices, and 43 percent of the decline in consumption. These results imply that the bank balance sheet amplification is larger on variables that depend on short-term debt, such as output, relative to those variables that depend on long-term debt,

bank balance sheet channel has a small effect during the boom period.

such as house prices.

An alternative method to compute this amplification is to shock the bank's net worth with the balance sheet losses of the benchmark economy at the peak of the boom, while keeping all parameters fixed at their boom level. This experiment shows that bank balance sheet losses account for 52 percent, 32 percent, and 45 percent of the declines in output, house prices, and consumption, respectively, which are not very different from the numbers we report above. Therefore, the change in the ordering of the decomposition have a minimal effect on the amplification arising from bank balance sheet deterioration during the bust.

Valuation versus Foreclosure Losses: Bank balance sheets deteriorate at the time of the bust for two reasons: the increase in foreclosures and the decline in mortgage prices (valuations). We decompose the relative importance of these two channels for the bank balance sheet amplification.[37] The sub columns "Valuation Losses" and "Foreclosures" in Table 3 report the relative contributions of only valuation losses or only foreclosure losses to the "BBS" amplification, which reveal that the losses in mortgage valuations are much more important than foreclosure losses (around 90 percent versus 10 percent, respectively), which is consistent with the evidence presented in IMF (2009).

While the relative contribution of the foreclosures might appear small, bear in mind that we are exclusively decomposing the effect of the credit supply shock. Consequently, this conclusion specifically pertains to the credit-supply-driven bust, which generates 60 percent of the increase in foreclosures. This does not suggest that foreclosures accounted for only 10 percent of bank balance sheet losses, as the actual foreclosure rate was larger during the crisis. As we will illustrate later in Section 5, the losses due to foreclosures are higher under shocks to the credit demand, such as housing demand or productivity shocks.

[37] The "foreclosure effect" is computed as solving the benchmark economy by not reflecting the changes in the market value of the existing mortgages on bank net worth at the time of the bust shock. The "valuation effect" is computed as solving the benchmark by not reflecting the foreclosure losses on bank net worth at the time of the bust shock.

Table 3: The Amplification through the Bank Balance Sheets (BBS)

Variables	Benchmark Bust Δ%	Contribution of BBS (%)	Contribution to BBS Effect (%)	
			Valuation Losses	Foreclosures
Output	–8.4	61	91	11
Consumption	–12.8	43	91	11
House Prices	–24.5	35	92	9

Notes: The "BBS" column reports the % contribution of the bank net worth deterioration to the bust. The "Valuation Losses" and "Foreclosures"columns report the contribution of each mechanisms to the BBS effect.

4.3 Distributional Implications

In this section, we show that our model's cross-sectional implications are also consistent with the recent evidence from detailed micro-level data analysis, some of which are argued to be inconsistent with the credit supply mechanism. Using a model with heterogeneous households allows us to show that credit supply shocks not only generate a boom-bust cycle in aggregates consistent with the data, but also have micro-level implications in line with the micro evidence, strengthening the argument for the importance of the credit supply channel for the US boom bust.

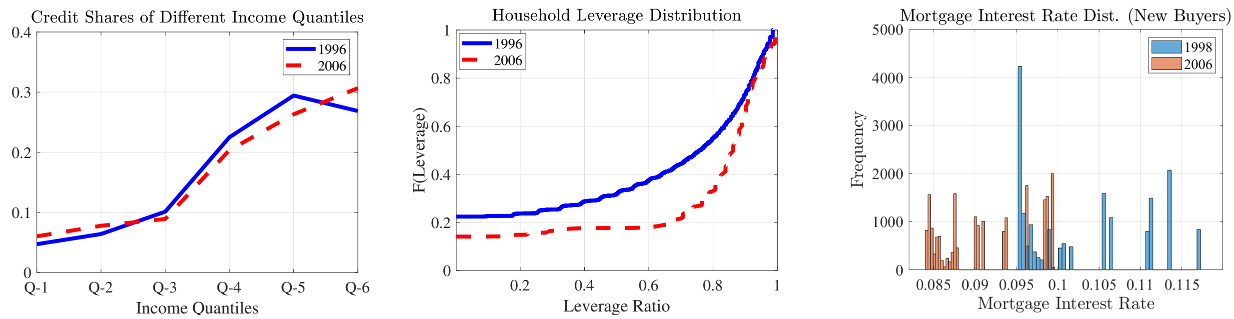
4.3.1 Credit Growth across Income Groups during the Boom Period

The initial findings in Mian and Sufi (2009) were mostly interpreted as a strong indication that the financial crisis may be a consequence of an unprecedented increase in lending to low-income and subprime borrowers.[38] However, Adelino et al. (2016), Albanesi et al. (2017), and Foote et al. (2016) find that credit grew uniformly across income groups during the boom period. These findings have been considered to be more consistent with the house price boom driven by expectations of capital gains (i.e., a credit demand channel rather than a credit supply channel).

In order to check whether our model is consistent with these more recent empirical findings, we first analyze how credit shares of each income quantile evolved during the boom

[38] Mian and Sufi (2016) point out that the credit supply view of the mortgage boom does not necessarily imply that individuals with the lowest-income or lowest credit score were responsible for the rise in aggregate household debt.

Figure 5: Cross-sectional Developments in Credit and Housing during the Boom



Notes: The graph plots several model-implied cross-sectional implications. The left panel plots the mortgage credit distribution across income quantiles in 1996 and 2006. The middle and right panels plot the CDF and the histogram of the down payment ratio and interest on mortgages in 1996 and at the peak of the boom in 2006.

period. The left panel of Figure 5 shows that the credit shares of each income quantile remained stable from 1996 to 2006, consistent with the evidence in Adelino et al. (2016), Albanesi et al. (2017), and Foote et al. (2016). The middle panel shows that household leverage is higher in 2006 than 1996 in the model, in part because the model generates endogenous increases in LTV ratios consistent with the data without appealing to exogenous changes in LTV limits.[39] The right panel shows that interest rates on mortgages declined from 1996 to 2006, as in the data (Favilukis et al. (2017)).

Finally, Adelino et al. (2016) further show that income growth and credit growth were positively related during the boom period. To find out whether our model is consistent with the data in this dimension as well, we regress an individual's credit growth on his/her age, asset, housing, and income growth using model-generated panel data. We restrict our sample to individuals who switch from renting to owning to focus on new mortgage originations, as in Adelino et al. (2016). Table 4 shows that income growth has a positive coefficient. Thus, overall, our model's implications are consistent with the evidence in Adelino et al. (2016).

[39] Keys et al. (2012) report that the average combined LTV of households increased by 15 percentage points during the crises, whereas our model implies a 13 percentage points increase.

Table 4: Change in Credit and Income Growth

Credit (2006)	Coefficient Estimates
Age	-0.002***
House	1.44***
Financial assets	-0.57***
Income (2006)/income(1996)	0.10***

Notes: This table presents the results from a regression of credit growth on income growth using model-generated panel data. "House" variable corresponds to the level of housing services that households consume. All the coefficients are significant at 1-percent.

4.3.2 The Roles of Household Leverage and Income on Default

An additional support for our framework comes from the determinants of mortgage default. The literature, so far, has identified two major factors that drive foreclosures: leverage and unemployment (Gerardi et al. (2008), Foote et al. (2010), Palmer (2015), and Indarte (2023)). We use model-generated panel data and estimate a linear regression model to analyze the determinants of default in the model. In particular, we are interested in the roles of household leverage prior to the bust and the decline in labor income from the peak of the boom to the bust.[40] We find that households with higher leverage during the boom period were more likely to default (Table 5). Similarly, a sharper decline in household labor income is associated with higher default.

5 Alternative Shocks

Other shocks have also been suggested as the driver of the boom-bust cycle around 2008. In this section, we compare our benchmark results with bank leverage shocks to the results with shocks to productivity and housing demand. To make the model dynamics comparable across shocks, we choose the size of these shocks to generate a similar-sized boom in house prices in all economies. Then, we revert the shocks to their initial steady-state values in the

[40]We investigate the role of decline in labor income since the model period is 2 years and unemployment durations are much shorter. As a result, increases in unemployment translates into declines in labor income during the 2-year model period.

Table 5: Determinants of Default in the Model

Default	Coefficient Estimates
Age	0.0026***
Financial assets	-0.0072***
Leverage	0.0192***
Income (bust)/income(boom)	-0.0260***

Notes: This table presents the results from a regression of default decision on several variables using model-generated panel data. Default takes a value of 1 if there is a default, 0 otherwise. Leverage is measured by mortgage debt divided by the house value. All the coefficients are significant at 1-percent.

bust. Because of difficulties in judging the path of the macro shocks, we do not aim to have similar-sized busts. We report our results in Figure 6. Overall, our results suggest that, while there are many similarities, there are also important differences in the model dynamics across different shocks.

One of the key differences between the leverage shock and the housing demand and productivity shocks is that the leverage shock primarily affects the credit supply while productivity and expectation shocks primarily affect the credit demand. For example, a reversal in housing demand in the bust lowers credit (mortgage) demand. The credit supply also declines since foreclosures worsen bank balance sheets. However, even though foreclosures increase more under the housing demand and productivity shocks than under the leverage shock, the equilibrium bank lending rate increases by much less.[41] As a result, mortgage values and hence bank net worth decline by much less under the housing demand and productivity shocks. Remember that, under the bank leverage shock, the decline in bank net worth is mostly driven by the decline in mortgage valuations. On the other hand, the valuation effect turns out to be less important than foreclosures under expectation and productivity shocks. Thus, with relatively smaller valuation effects, these shocks cannot generate a large deterioration in bank balance sheets.

Figure 6 shows that output increases with productivity and leverage shocks but stays

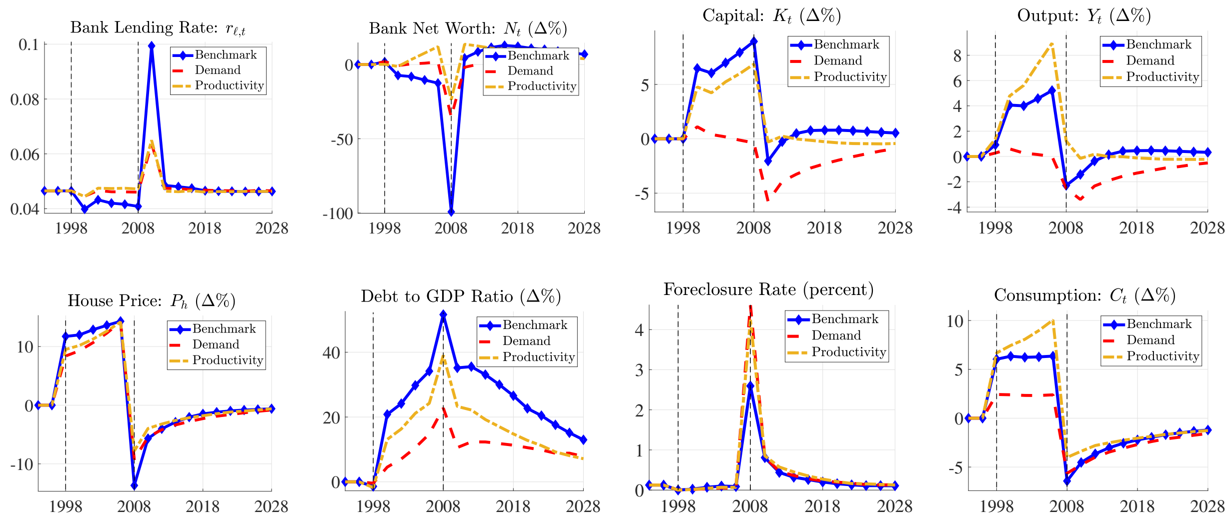
[41] Overall, the bank lending rate declines during the boom and increases during the bust with the leverage shock, consistent with the evidence reported in Adrian et al. (2013) and Gilchrist and Zakrajšek (2012b).

flat with the housing demand shock during the boom. This is mainly because, with the housing demand shock, capital stock does not increase during the boom as households use some part of their capital to purchase a house. A similar but less stark difference arises in the consumption dynamics. While consumption increases strongly with productivity and leverage shocks during the boom, it barely increases with the housing demand shock. This finding also suggests that the relationship between house prices and consumption may critically depend on the shock that generates the cycle. It will be low if the boom is generated by a housing demand shock and will be high with productivity and leverage shocks.

The dynamics of the debt-to-GDP ratio, as well, differ across shocks, increasing by 52 percent with the leverage chock, 39 percent with the productivity shock, and 23 percent with the expectation shock during the boom. In the case of the leverage shock, a lower bank lending rate and around a 6 percent labor income growth drive the household debt. Strong income growth (close to 9 percent) supports housing and credit demand in the productivity shock case. With the housing demand shock, neither interest rates nor income growth increases demand. Household debt increases only because of the house price increase. As a result, credit growth remains relatively low during the housing boom. House prices rise close to 14 percent in all cases but decline less with the productivity and housing demand shocks.

We contend that all of these shocks might have contributed to the housing boom and the bust. However, our results suggest that the leverage shock is the strongest candidate among the three. It generates reasonable fluctuations in almost all variables together with severe banking crises during the bust period.

Figure 6: Credit Supply versus Credit Demand Shocks



Notes: The graph plots the dynamics of the model with three different shocks: bank leverage (benchmark), productivity, and demand (housing). The sizes of shocks are chosen to match the same increase in the housing prices.

6 Conclusions

In this paper, we developed a heterogeneous-agent model that features interactions between household, firm, and bank balance sheets and is consistent with important cross-sectional facts as well as the dynamics of key aggregate variables around the 2008 boom-bust cycle in the US. Shocks to bank leverage contribute significantly to the boom-bust in house prices, output, wages, and consumption. The feedback from the bank balance sheets into household and firm borrowing rates, the latter affecting household labor income, plays an important role in the amplification of shocks. Overall, our results show that the change in credit supply—whether it is due to the exogenous shocks to bank leverage or an endogenous response to the bank balance sheet deterioration during the bust—is an important contributor to the boom-bust in the housing market and overall economy.

To ensure the clarity of our analysis, we have omitted several factors that might be relevant for the quantitative results. Notably, we have not incorporated monetary or fiscal policy responses to the crises, as their effectiveness heavily depends on the anticipated trajectory of such policies. Furthermore, we have not considered the feedback from consumption to output, as in HANK models, which could potentially increase the influence of house prices on output. Additionally, we have not accounted for heterogeneity among banks and bank defaults. We intend to explore these extensions in future research.

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Internet Appendix

A Data

GDP Real gross domestic product per capita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Consumption Real personal consumption expenditures percapita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Labor income Total wages and salaries (Not Seasonally Adjusted Annual Rate from FRED) divided by working-age population and then divided by the price index for non-durable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use annual data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2018 from this trend.

Hours per person Hours of Wage and Salary Workers on Nonfarm Payrolls (From FRED, Total, Billions of Hours, Quarterly, Seasonally Adjusted Annual Rate) divided by Working Age Population (From FRED, Aged 15-64: All Persons for the United States, Persons, Quarterly, Seasonally Adjusted).

Investment Private Nonresidential Fixed Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Homeownership Rate Census Bureau Homeownership rate for the U.S. (Table 14) and by age of the householder (Table 19). Housing Vacancies and Homeownership (CPS/HVS) - Historical Tables.

House Prices House Price Index for the entire US (Source: Federal Housing Finance Agency) divided by the price index for nondurable consumption (line 6 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percent deviation of the data from 1985 to 2018 from this trend. To obtain the changes relative to GDP, we divide the real house price index by the real GDP series.

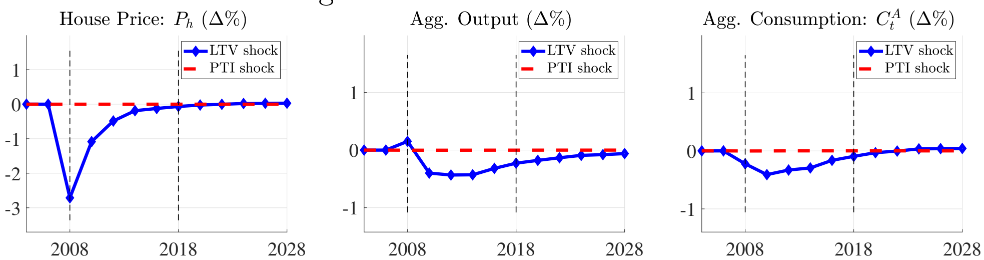
House Rent-Price Ratio Rents (Bureau of labor Statistics Consumer Price Index for All Urban Consumers: Rent of primary residence) divided by nominal house prices.

Household Leverage Home Mortgage Liabilities divided by Owner Occupied Housing Real Estate at Market Value. Source: Flow of Funds B.101 Balance Sheet of Households and Nonprofit Organizations.

B Extensions and Robustness

B.1 Shocks to Loan-to-Value and Payment-to-Income Limits

Figure 7: LTV and PTI Busts



Notes: This figure plots the model dynamics where LTV and PTI constraints tighten unexpectedly. The LTV constraint tightens from 100 to 80 percent, and the PTI constraint tightens to 25 percent (no PTI constraints in the benchmark) with a persistence of 0.6. We focus on the bust since relaxation of the constraints does not generate significant booms.

B.2 Robustness with Respect to Parameters

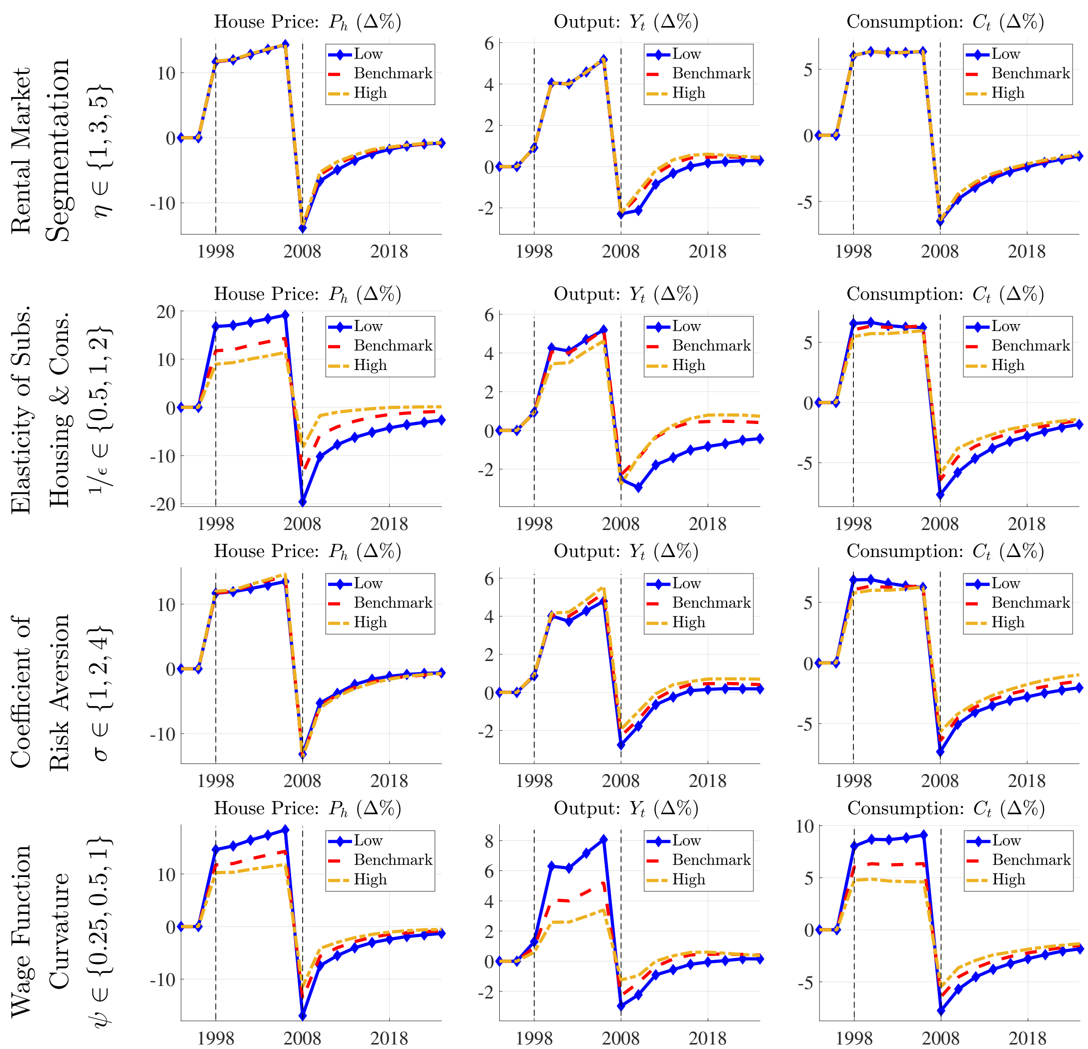
For all robustness analysis, we recalibrate the initial steady state of the model. Then we feed into the economy the same leverage shock from our benchmark. The boom-bust dynamics look almost identical for different values of η, ϵ, and σ as shown in Figure 8.

The response of hours (hence, of output and labor income) to changes in r_ℓ depends on the labor utilization curvature ψ. We set $\psi = 0.5$ so that the employment response to changes in credit spreads in our benchmark is consistent with the data. For higher values of ψ, the response of employment to changes in r_ℓ becomes smaller; hence, the importance of the changes in credit supply (both exogenous and endogenous) for the boom-bust. However, the role of credit supply stays large.

B.3 Boom-Bust Generated by Housing Demand and Leverage Shocks

In this extension, we use our model to quantify the importance of credit supply versus housing demand shocks for the boom-bust cycle as observed in the US around 2008. To study the boom-bust episode, we calibrate the model to match several US data moments—most

Figure 8: The boom-bust dynamics with alternative parameterizations



importantly, the ones regarding household and bank balance sheets—before 1998 (as we do in our benchmark analysis in the main text). We then give two subsequent and unexpected shock pairs: leverage shocks to bank balance sheets and housing demand shocks. First, in 1998, banks start increasing their leverage over time along with an increasing housing demand. We calibrate the size of the boom leverage shock such that the changes in the banks' book leverage match the data during the boom. We then calibrate the housing demand shock to match the remainder of the house price boom in the data, which is beyond what is generated by the bank leverage shock. Second, in 2008, both shocks revert to their initial steady-state values. The calibration of the leverage shock is essentially the same as the one in benchmark economy, though calibrated value differs slightly.

Housing Demand Shock: The boom leverage shock generates part of the housing boom. We generate the remainder of the housing boom with a housing demand shock in the spirit of Kaplan et al. (2020). We assume that homeowners receive a premium χ_t in $u\left(\left(1+\chi_t\right)c, \left(1+\chi_t\right)s\right)$ with $\chi_t = 0$ in the initial steady state. We calibrate the increase in χ_t to match the remainder of the house price boom in the data so that both shocks together generate a 28 percent increase in house prices during the boom. Then, χ_t unexpectedly reverts to its initial steady-state value of zero in 2008. Similar to the leverage shock, starting in 1998, we impose a linear change in the homeownership premium parameter.

Government Interventions at the Bust: Even though both shocks return to their initial steady state levels in 2008, the model economy experiences a large bust, in which bank net worth becomes negative in the absence of any government interventions. This result suggests that the government interventions were necessary to keep the banking sector afloat in the crises. In order to incorporate interventions into our framework that are consistent with the actual experience, we assume that the government borrows from international investors at the rate r in 2008 to finance bailouts to households and banks and rolls over its debt until 2038, after which it increases the labor income tax to balance the government budget in the long run.[42] For the household bailout, we assume that the government pays banks for the portion of a household's debt that is above a threshold leverage ratio. We choose the threshold to match the 3.6 percent increase in the foreclosure rate during the bust. The calibrated threshold is 120 percent, which turns out to be close to the one used by the Home Affordable Modification Program (HAMP).[43] For the bank bailout, we assume that the government injects equity into banks so that the bank net worth ω_t declines by 67 percent in 2008, consistent with the decline in market equity of bank holding companies reported in Begenau et al. (2019, Figure 3, left panel). With this intervention, the bank collateral premium or equivalently the credit spread $(r_\ell - r)$, the measure of the distress in the banking system, increases by 5.6 percentage points. Finally, the total bailout amount in the model is 5.4 percent of GDP.[44]

B.3.1 The Importance of Leverage and Housing Demand Shocks

Changes in credit supply and housing demand have been proposed as two competing explanations for the boom-bust cycle in the US housing market. In this section, we quantify the importance of the bank leverage shock (one source of credit supply change) and the housing demand shock to the boom-bust. For this, we solve the equilibrium transition of the model with only one "boom" shock (bank leverage or housing demand) and compute the size of the boom generated by that shock only. We report the results of these exercises under "Boom/Only/Δx_{LS}" and "Boom/Only/Δx_{HDS}" columns in Table 6. We also report the changes in variables from our benchmark with two shocks under "Boom/Benchmark/$\Delta x_{(\text{LS+ HDS})}$" for

[42]The particular choice of the year after which the labor income tax is increased does not affect the boom-bust dynamics we report in the paper as long as it is in the distant future. We have also experimented with no tax increase as if the bailout amount is a windfall to check the sensitivity of our results. Again, we did not discern any noticeable effect on the boom-bust.

[43]HAMP provided debt forgiveness and payment modification to households with leverage ratios above 115 percent. See Ganong and Noel (2020) for the details of these programs.

[44]The US Congress approved \$700 billion as part of the Troubled Asset Relief Program (TARP) in 2009. However, only \$475 billion of this amount was actually used. Private sector GDP in 2009 was \$11.8 trillion (\$14.7 trillion total value added minus \$2.9 trillion value added by the government). The amount of \$700 (\$475) billion is 6 (4) percent of GDP. Even though the bailout amount is larger in the model than in the data, remember that we have only two fiscal interventions and abstract from the monetary policy response since the effectiveness of monetary interventions depends crucially on people's expectations about the future policy path, which makes our decomposition exercises blurry and significantly more complicated.

Table 6: Contributions of shocks to leverage (LS) and housing demand (HDS) to boom-bust

	Boom			Bust		
	Benchmark (two shocks)	Only		Benchmark	Only	
	$\Delta x_{(\text{LS + HDS})}$	Δx_{LS}	Δx_{HDS}	$\Delta x_{(\text{LS + HDS +Bailout})}$	Δx_{LS}	Δx_{HDS}
Bank lending rate (Δr_ℓ)	-0.5	–0.5	0.0	4.5	5.2	1.7
Output ($\%\Delta Y$)	5.0	4.4	0.2	–7.5	-6.5	-2.6
Labor income ($\Delta\% w$)	5.4	5.4	-0.5	-12.1	-13.7	-4.4
Consumption ($\%\Delta C$)	7.8	5.2	2.4	-14.7	-10.7	-8.0
House price ($\%\Delta p_h$)	28.0	12.3	13.1	-32.1	-19.3	-21.7
Foreclosure rate (ΔFr)	0.0	-0.1	-0.1	3.6	0.7	6.4

Note: The table reports the results where we decompose the role of leverage and housing demand shocks. "LS" refers to leverage shock, "HDS" refers to housing demand shock. "Only" columns report the results with only one of the shocks. The percent decline in the bust is calculated with respect to the value at the peak of the boom.

comparison.

Focusing on the boom, we find that the leverage shock by itself generates a 12.3 percent increase in house prices and virtually all of the decline in the bank lending rate and most of the increase in wages and output. Consumption, driven by house prices and wages, increases by 5.2 percent. Thus, overall the contribution of the leverage shock to the boom remains close to our benchmark results.

The housing demand shock generates a slightly larger increase in house prices (13.1 percent). However, it does not significantly affect the bank lending rate, and has small effects on output and consumption (0.2 and 2.4 percent, respectively).

To assess the contributions of each of these shocks to the bust, we hit the economy with one bust shock (bank leverage or housing demand) while the economy is at the peak of the benchmark boom generated by both shocks (the "Bust" column in Table 6). We find large effects for both leverage and housing demand shocks during the bust. The bank lending rate increases by 5.2 and 1.7 percentage points, and output declines by 6.5 and 2.6 percent on impact, with leverage and housing demand shocks, respectively. The housing demand shock generates a slightly bigger decline in house prices (21.7 versus 19.3) and a larger increase in the foreclosure rate (6.4 versus 0.7 percentage points) relative to the leverage shock. The disproportionately larger impact of the housing demand shock on foreclosures is mainly because it reduces the ownership benefit and makes foreclosure a more attractive option for a given house price decline. Finally, the decline in consumption is driven by the declines in house prices and income, and the leverage shock generates a 10.7 percent decline while the housing demand shock generates a 8.0 percent decline.

The housing demand shock affects all aggregates more during the bust than the boom because the decline in house prices increases foreclosures and hurts bank balance sheets. The resulting decline in credit supply causes the equilibrium bank lending rate to increase by 1.7 percentage points and labor income to decline by 4.4 percent. Thus, the role of the housing demand shock is amplified by its effect on the bank balance sheet deterioration.

We want to conclude this section with a remark. It might be confusing to observe that the leverage shock by itself generates a 6.5 percent decline and the housing demand shock generates a 2.6 percent decline in output, while the actual decline in the data is only 5.3 percent (a similar argument applies to other variables as well). However, it is important to note that there are bailouts during the recession. Thus, our result suggests that the bailouts significantly counteracted the contribution of these shocks.[45]

[45]We do not report the amount of bust mitigated by the bailouts. In principle, we could compute the amount of bust generated by both shocks without bailouts, call it $\Delta x_{\text{LS+HDS}}$. Then the difference between the benchmark and the bust generated by this exercise would be the effect of bailouts: $\Delta x_{\text{bailout}} = \Delta x_{(\text{LS + HDS +Bailout})} - \Delta x_{(\text{LS+HDS})}$. However, as we noted earlier, the bank net worth becomes negative in the bust in the absence of bailouts. Figuring out how the economy evolves from that point generates further complications to our analyses. Thus, we cannot obtain $\Delta x_{\text{LS+HDS}}$. An alternative is to compute the effect of the bailouts as $\Delta x_{\text{bailout}} = \Delta x_{(\text{LS + HDS +Bailout})} - (\Delta x_{\text{LS}} + \Delta x_{\text{HDS}})$, which can be read as the residual from Table 6. To the extent that $\Delta x_{\text{LS}} + \Delta x_{\text{HDS}} = \Delta x_{\text{LS+HDS}}$, these two exercises would give similar results. However, this equality does not necessarily hold since the ordering of decomposition matters.

C Value Functions for Households

C.1 Active Renters

An active renter has two choices: continue to rent or purchase a house, that is, $V^r = \max\left\{V^{rr}, V^{rh}\right\}$ where V^{rr} is the value function if she decides to continue renting and V^{rh} is the value function if she decides to purchase a house. If she decides to continue to rent, she chooses rental unit size s at price p_r per unit, makes her consumption and saving choices, and remains as an active renter in the next period. After purchasing a house, she begins the next period as a homeowner. The value function of an active renter who decides to remain as a renter is given by

$$V_j^{rr}(a, z) = \max_{c,s,a' \geq 0} \left\{u(c, s) + \beta E V_{j+1}^r(a', z')\right\} \tag{6}$$

subject to

$$c + \frac{a'}{1 + r_k} + p_r s = y(j, z) + a,$$

where a is the beginning-of-period financial wealth, $p_r s$ is the rental payment, r_k is the return to savings, and w is the wage rate per efficiency unit of labor. The expectation operator is over the income shock z'.

C.2 Inactive Renters

Inactive renters are not allowed to purchase a house because of their default in previous periods. However, they can become active renters with probability π. Since they cannot buy a house; they only make rental size, consumption, and saving decisions. The value function of an inactive renter is given by

$$V_j^e(a, z) = \max_{c,s,a' \geq 0} \left\{u(c, s) + \beta\left[\pi E V_{j+1}^r(a', z') + (1 - \pi) E V_{j+1}^e(a', z')\right]\right\} \tag{7}$$

subject to

$$c + \frac{a'}{1 + r_k} + p_r s \quad = \quad y(j, z) + a.$$

C.3 Homeowners

The options of a homeowner are: 1) stay as a homeowner, 2) refinance, 3) sell the current house (become a renter or buy a new house), or 4) default. The value function of an owner is given as the maximum of these four options, that is, $V^h = \max\left\{V^{hh}, V^{hf}, V^{hr}, V^{he}\right\}$, where V^{hh} is the value of staying as a homeowner, V^{hf} is the value of refinancing, V^{hr} is the value of selling, and V^{he} is the value of defaulting (being excluded from the ownership option).

A stayer makes a consumption and saving decision given his income shock, housing, mortgage debt, and assets. Therefore, the problem of the stayer can be formulated as follows:

$$V_j^{hh}(a, h, d, z) = \max_{c, a' \geq 0} \left\{u(c, h) + \beta E V_{j+1}^h(a', h, d', z')\right\} \tag{8}$$

subject to

$$\begin{aligned} c + \delta_h p_h h + \frac{a'}{1 + r_k} + m &= y(j, z) + a \\ d' &= (d - m)(1 + r_\ell), \end{aligned}$$

where m is the mortgage payment following the standard amortization schedule computed at the bank lending rate r_ℓ.

The second choice for the homeowner is to refinance, which also includes prepayment. Refinancing requires paying the full balance of any existing debt and getting a new mortgage. We assume that refinancing is subject to the same transaction costs as new mortgage originations. So, we can formulate the problem of a refinancer as

$$V_j^{hf}(a, h, d, z) = \max_{c, d', a' \geq 0} \left\{u(c, h) + \beta E V_{j+1}^h(a', h, d', z')\right\} \tag{9}$$

subject to

$$c + d + \delta_h p_h h + \varphi_f + \frac{a'}{1 + r_k} = y(j, z) + a + d'\left(q^m(d'; a, h, z, j) - \varphi_m\right).$$

The third choice for the homeowner is to sell the current house and either stay as a renter or buy a new house. Selling a house is subject to a transaction cost that equals fraction φ_s of the selling price. Moreover, a seller has to pay the outstanding mortgage debt, d, in full to the lender. A seller, upon selling the house, can either rent a house or buy a new one. Her problem is identical to a renter's problem. So, we have

$$V_j^{hr}(a,h,d,z) = V_j^r(a + p_h h(1-\varphi_s) - d, z).$$

The fourth possible choice for a homeowner is to default on the mortgage, if she has one. A defaulter has no obligation to the bank. The bank seizes the house, sells it on the market, and returns any positive amount from the sale of the house, net of the outstanding mortgage debt and transaction costs, back to the defaulter. For the lender, the sale price of the house is assumed to be $(1-\varphi_e)\,p_h h$. Therefore, the defaulter receives $\max\{(1-\varphi_e)\,p_h h - d, 0\}$ from the lender. The defaulter starts the next period as an active renter with probability π. With probability $(1-\pi)$, she stays as an inactive renter. The problem of a defaulter becomes the following:

$$V_j^{he}(a,d,z) = \max_{c,s,a'\geq 0}\left\{u(c,s) + \beta E\left[\pi V_{j+1}^r(a',z') + (1-\pi)\,V_{j+1}^e(a',z')\right]\right\} \tag{10}$$

subject to

$$c + \frac{a'}{1+r_k} + p_r s = a + y(j,z) + \max\{(1-\varphi_e)\,p_h h - d, 0\}.$$

The problem of a defaulter is different from the problem of a seller in two ways. First, the defaulter receives $\max\{(1-\varphi_e)\,p_h h - d, 0\}$ from the housing transaction, whereas a seller receives $(1-\varphi_s)\,p_h h - d$. We assume that the default cost is higher than the sale transaction cost, that is, $\varphi_e > \varphi_s$, and the defaulter receives less than the seller as long as $(1-\varphi_s)\,p_h h - d \geq 0$ (i.e., the home equity net of the transaction costs for the homeowner is positive). Second, a defaulter does not have access to the mortgage in the next period with some probability. Such an exclusion lowers the continuation utility for a defaulter. In sum, since defaulting is costly, a homeowner will choose to sell the house instead of defaulting as long as $(1-\varphi_s)\,p_h h - d \geq 0$ (i.e., net home equity is positive). Hence, negative equity is a necessary condition for default in the model. Therefore, in equilibrium, a defaulter gets nothing from the lender.

D Firm's Problem

The firm's first-order conditions are given as

$$\begin{aligned}
\alpha \mathbb{Z}_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha-1} &= r_{k,t} + \delta \\
(1-\alpha)\,\mathbb{Z}_t u_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= (1+\mu r_{\ell,t+1}) \left(\bar{w}_t + \vartheta \frac{u_t^{1+\psi}}{1+\psi}\right) \\
(1-\alpha)\,\mathbb{Z}_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= (1+\mu r_{\ell,t+1})\,\vartheta u_t^{\psi}.
\end{aligned}$$

E Rental Companies

The objective of the company is to maximize its total market value:

$$V_t^{rc}\left(H_{t-1}^r\right) = \max_{H_t^r} \frac{p_t^h (1-\delta) H_{t-1}^r - p_t^h H_t^r - \frac{\eta p_t^h \left(H_t^r - H_{t-1}^r\right)^2}{2} + (p_t^r - \kappa) H_t^r + V_{t+1}^{rc}\left(H_t^r\right)}{1 + r_{k,t}},$$

which leads to the following first-order condition:

$$p_t^r = \kappa + p_t^h + \eta p_t^h \left(H_t^r - H_{t-1}^r\right) - \frac{(1-\delta_h)\, p_{t+1}^h + \eta p_{t+1}^h \left(H_{t+1}^r - H_t^r\right)}{1 + r_{k,t+1}}.$$

F Characterization of the Bank's Problem

In this section, we will provide proofs for the characterization of the bank's problem. We will start with the steady-state value functions and decision rules and continue obtaining value functions in the transition by iterating backward from the steady state. The bank's problem is given as

$$\Psi_t\left(L_t, B_t\right) = \max_{B_{t+1}, L_{t+1}, c_t^B} \left\{\log\left(c_t^B\right) + \beta_L \Psi_{t+1}\left(L_{t+1}, B_{t+1}\right)\right\}$$

subject to

$$\begin{aligned} c_t^B + L_{t+1} &= \left(1 + r_{\ell,t}\right) L_t - \left(1 + r_t\right) B_t + B_{t+1} \\ \Psi_{t+1}\left(L_{t+1}, B_{t+1}\right) &\geq \tilde{\Psi}_{t+1}^D\left(\xi\left(1 + r_{\ell,t+1}\right) L_{t+1}\right), \end{aligned}$$

where $\tilde{\Psi}_t^D\left(W\right) = \max_{W'} \log\left(W - W'\right) + \beta_L \tilde{\Psi}_{t+1}^D\left((1 + r_{t+1})W'\right).$

F.1 Steady State with $r_\ell > r$

We will characterize the case $r_\ell > r$ and leave the cases for $r_\ell \leq r$ for brevity. We will start with the value function of the bank when it defaults. Since the bank can steal a fraction ξ of assets after the return has been realized and can continue saving at interest rate r, the bank's problem in the period of default is given as

$$\tilde{\Psi}^D\left(\xi L'\right) = \max_{s'} \log\left(\xi L' - W'\right) + \beta_L \Psi^D\left((1 + r)W'\right),$$

and after default, it becomes

$$\tilde{\Psi}^D\left(W\right) = \max_{s'} \log\left(W - W'\right) + \beta_L \Psi^D\left((1 + r)W'\right).$$

Lemma 1. $\tilde{\Psi}^D\left(W\right)$ *is given as*

$$\tilde{\Psi}^D\left(W\right) = \frac{1}{1 - \beta_L}\log(W) + \frac{\beta_L}{(1 - \beta_L)^2}\log(\beta_L(1 + r)) + \frac{\log(1 - \beta_L)}{1 - \beta_L}.$$

The bank's problem in the no-default state solves

$$\Psi(L,B)=\max_{L',B'}\log\left((1+r_\ell)L-(1+r)B+B'-L'\right)+\beta_L\Psi(L',B')$$

subject to

$$\Psi(L',B')\geq\tilde{\Psi}^D\left(\xi(1+r_\ell)L'\right).$$

Proposition 1. *The solution to the bank's problem is given as follows:*

1. Value function:

$$\begin{aligned}\Psi(L,B) &= \frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right)\\ &+ \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r)(1+r_\ell)\beta_L\phi}{1+r-(1+r_\ell)(1-\phi)}\right)+\frac{\log(1-\beta_L)}{1-\beta_L}.\end{aligned}$$

2. The no-default constraint can be written as

$$(1+r_\ell)(1-\phi)L'\geq(1+r)B'$$

   where ϕ is given as

$$\phi=\xi^{1-\beta_L}\left(\frac{1+r}{1+r_\ell}-(1-\phi)\right)^{\beta_L}.$$

3. The bank's solution satisfies the following expression regardless of whether or not the no-default constraint binds:

$$L'-B'=\beta_L\left((1+r_\ell)L-(1+r)B\right).$$

4. The decision rules when the no-default constraint is binding (if $r_\ell>r$):

$$\begin{aligned}L' &= \frac{(1+r)}{1+r-(1-\phi)(1+r_\ell)}\beta_L\left((1+r_\ell)L-(1+r)B\right)\\ B' &= \frac{(1-\phi)(1+r_\ell)}{1+r-(1-\phi)(1+r_\ell)}\beta_L\left((1+r_\ell)L-(1+r)B\right).\end{aligned}$$

Proof. (Proposition 1) We will use the expressions for value functions and verify the claims above. First, drive the capital requirement constraint:

$$\Psi\left(L', B'\right) \geq \tilde{\Psi}^D\left(\xi(1+r_\ell)L'\right).$$

$$\frac{1}{1-\beta_L}\log\left((1+r_\ell)L'-(1+r)B'\right)+\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right) \geq \frac{1}{1-\beta_L}\log(\xi(1+r_\ell)L')+\frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r)),$$

where ϕ' is the capital requirement constraint in the next period. The expression above gives

$$\log\left(\frac{(1+r_\ell)L'-(1+r)B'}{\xi(1+r_\ell)L'}\right) \geq \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta\left((1+r)-(1+r_\ell)(1-\phi')\right)}{(1+r_\ell)\beta_L\phi'}\right)$$

$$\frac{(1+r_\ell)L'-(1+r)B'}{(1+r_\ell)L'} \geq \xi\left(\frac{((1+r)-(1+r_\ell)(1-\phi'))}{(1+r)\phi'}\right)^{\frac{\beta_L}{1-\beta_L}}.$$

We will show below that the solution of ϕ' is the fixed point of

$$\phi = \xi\left(\frac{((1+r)-(1+r_\ell)(1-\phi'))}{(1+r)\phi'}\right)^{\frac{\beta_L}{1-\beta_L}}.$$

Then this constraint can be written as

$$(1+r_\ell)\left(1-\phi\right)L' \geq (1+r)\,B'.$$

Now, we can solve the bank's problem:

$$\begin{aligned}
\Psi\left(L,B\right) &= \max_{L',B'}\log\left((1+r_\ell)L-(1+r)B+B'-L'\right)+\beta_L\Psi\left(L',B'\right)\\
&= \max_{L',B'}\log\left((1+r_\ell)L-(1+r)B+B'-L'\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L'-(1+r)B'\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

subject to

$$(1+r_\ell)(1-\phi)L' \geq (1+r)B'.$$

Imposing the balance sheet constraint, we obtain

$$\begin{aligned}
\Psi(L,B) &= \max_{L',B'} \log\left((1+r_\ell)L-(1+r)B+\frac{(1+r_\ell)(1-\phi)L'}{1+r}-L'\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L'-(1+r)\frac{(1+r_\ell)(1-\phi)L'}{1+r}\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

$$\begin{aligned}
\Psi(L,B) &= \max_{L'} \log\left((1+r_\ell)L-(1+r)B-\frac{(1+r)-(1+r_\ell)(1-\phi)}{1+r}L'\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)\phi L'\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}$$

The first-order condition is

$$\frac{\frac{(1+r)-(1+r_\ell)(1-\phi)}{1+r}}{(1+r_\ell)L-(1+r)B-\frac{(1+r)-(1+r_\ell)(1-\phi)}{1+r}L'}=\frac{\beta_L}{1-\beta_L}\frac{1}{L'},$$

which gives

$$\begin{aligned}
L' &= \frac{\beta_L(1+r)}{(1+r)-(1-\phi)(1+r_\ell)}\left((1+r_\ell)L-(1+r)B\right)\\
B' &= \frac{\beta_L(1-\phi')(1+r_\ell)}{(1+r)-(1-\phi')(1+r_\ell)}\left((1+r_\ell)L-(1+r)B\right).
\end{aligned}$$

Given these decision rules, the value function is given by

$$
\begin{aligned}
\Psi(L,B) &= \frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r_\ell)\,(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)\,(1-\phi')}\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right)+\frac{\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}
$$

Equating this expression to our initial guess,

$$
\frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right)+\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi)}\right)+\frac{\log(1-\beta_L)}{1-\beta_L},
$$

we obtain

$$
\begin{aligned}
\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi)}\right) &= \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r_\ell)\,(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)\,(1-\phi)}\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right),
\end{aligned}
$$

which gives

$$
\frac{\phi}{(1+r)-(1+r_\ell)(1-\phi)}=\frac{\phi'}{(1+r)-(1+r_\ell)(1-\phi')}.
$$

Since these expressions are monotone (and declining) in ϕ, they imply that $\phi=\phi'$. By imposing this into

$$
\phi=\xi\left(\frac{1+r-(1+r_\ell)(1-\phi')}{(1+r)\phi'}\right)^{\frac{\beta_L}{1-\beta_L}}.
$$

we obtain

$$
\phi=\xi^{1-\beta_L}\left(\frac{1+r-(1+r_\ell)(1-\phi)}{(1+r)}\right)^{\beta_L}.
$$

□

F.2 Transition

Assume that the last period of the transition is period T and the economy is in steady state with r_ℓ and r from period $T+1$ and onward. The following proposition characterizes the bank's solution in the transition, where all prices $r_{\ell,t}$ and r_t are potentially changing.

Proposition 2. *The solution to the bank's problem is given as follows:*

1. The value function:

$$\Psi_t\left(L_t, B_t\right) = \frac{1}{1-\beta_L}\log\left((1+r_{\ell,t})L_t - (1+r_t)B_t\right) + \Omega_t + \frac{\log(1-\beta_L)}{1-\beta_L},$$

   where

$$\Omega_t = \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_{t+1}(1+r_{t+1})1+r_{\ell,t+1}}{1+r_{t+1}-(1-\phi_{t+1})1+r_{\ell,t+1}}\right) + \beta_L\Omega_{t+1};$$

$$\Omega_T = \Omega = \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r)\left(1+r_\ell\right)}{1+r-(1-\phi)\left(1+r_\ell\right)}\right);$$

$$\phi_t = \xi^{1-\beta_L}\left(\frac{1+r_{t+1}}{1+r_{\ell,t+1}} - (1-\phi_{t+1})\right)^{\beta_L};$$

   and

$$\phi_T = \phi.$$

2. The no-default constraint in period t can be written as

$$\left(1+r_{\ell,t+1}\right)\left(1-\phi_{t+1}\right)L_{t+1} \geq \left(1+r_{t+1}\right)B_{t+1}.$$

3. The bank's solution satisfies the following expression regardless of whether or not the no-default constraint binds:

$$L_{t+1} - B_{t+1} = \beta_L\left((1+r_{\ell,t})L_t - (1+r_t)B_t\right).$$

4. The decision rules when the no-default constraint is binding (if $r_{\ell,t+1} > r_{t+1}$):

$$\begin{aligned}
L_{t+1} &= \frac{\beta_L(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right) \\
B_{t+1} &= \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right).
\end{aligned}$$

5. The decision rules when the no-default constraint is not binding (if $r_{\ell,t+1} \leq r_{t+1}$):

$$B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right)\right] & \text{if } r_{\ell,t+1} = r_{t+1} \\ 0 & \text{if } r_{\ell,t+1} < r_{t+1} \end{cases}$$

and

$$L_{t+1} = B_{t+1} + \beta_L\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right).$$

Proof. We are going to solve the problem backward starting from period T.

Period T:

$$\begin{aligned}
\Psi_T(L_T, B_T) &= \max_{L_{T+1},B_{T+1}} \log\left((1+r_{\ell,T})L_T-(1+r_T)B_T-(L_{T+1}-B_{T+1})\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L_{T+1}-(1+r)B_{T+1}\right) \\
&+ \left(\frac{\beta_L}{1-\beta_L}\right)^2 \log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right) + \frac{\beta_L}{1-\beta_L}\log(1-\beta_L) \\
&\text{s.t.} \\
(1-\phi)(1+r_\ell)L_{T+1} &\geq (1+r)B_{T+1}.
\end{aligned}$$

The decision rules of this problem are given as

$$\begin{aligned}
L_{T+1} &= \frac{\beta_L(1+r)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\
B_{T+1} &= \frac{\beta_L(1-\phi)(1+r_\ell)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\
L_{T+1}-B_{T+1} &= \beta_L\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\
(1+r_\ell)L_{T+1}-(1+r)B_{T+1} &= \frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right)
\end{aligned}$$

which give

$$\Psi_T\left(L_T, B_T\right) = \frac{1}{1-\beta_L}\log\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) + \frac{\beta_L}{\left(1-\beta_L\right)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).$$

The value function when the bank defaults is

$$\tilde{\Psi}_T^D\left(\xi(1+r_{\ell,T})L_T\right)=\frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T})L_T\right)+\frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r))+\frac{\log(1-\beta_L)}{1-\beta_L}.$$

The no-default condition in period T can be written as

$$(1-\phi_T)(1+r_{\ell,T})L_T \geq (1+r_T)B_T,$$

where

$$\phi_T=\xi^{1-\beta_L}\left(\frac{1+r}{1+r_\ell}-(1-\phi)\right)^{\beta_L}.$$

Period $T-1$:

$$\begin{aligned}\Psi_{T-1}\left(L_{T-1}, B_{T-1}\right) &= \max_{L_T,B_T}\log\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}-(L_T-B_T)\right)\\ &+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right)\\ &+ \left(\frac{\beta_L}{1-\beta_L}\right)^2\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{\beta_L}{1-\beta_L}\log(1-\beta_L)\\ \text{s.t.}&\\ (1-\phi_T)(1+r_{\ell,T})L_T &\geq (1+r_T)B_T.\end{aligned}$$

The decision rules for this problem are given as

$$\begin{aligned}
L_T &= \frac{\beta_L(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right) \\
B_T &= \frac{\beta_L(1-\phi_T)(1+r_{\ell,t})}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right) \\
L_T-B_T &= \beta_L\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right) \\
(1+r_{\ell,t})L_T-(1+r_T)B_T &= \frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right),
\end{aligned}$$

which give

$$\begin{aligned}
\Psi_{T-1}\left(L_{T-1},B_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).
\end{aligned}$$

The value function when the bank defaults is

$$\begin{aligned}
\tilde{\Psi}^D_{T-1}\left(\xi(1+r_{\ell,T-1})L_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T-1})L_{T-1}\right)+\frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r^D_T)) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log(\beta_L(1+r^D))+\frac{\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}$$

The no-default condition in period $T-1$ can be written as

$$(1-\phi_{T-1})(1+r_{\ell,T-1})L_{T-1}\geq(1+r_{T-1})B_{T-1},$$

where

$$\phi_{T-1}=\xi^{1-\beta_L}\left(\frac{1+r_T}{1+r_{\ell,t}}-(1-\phi_T)\right)^{\beta_L}.$$

Period $T-2$**:**

$$\begin{aligned}
\Psi_{T-2}\left(L_{T-2},B_{T-2}\right) &= \max_{L_{T-1},B_{T-1}} \log\left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}-(L_{T-1}-B_{T-1})\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right) \\
&+ \frac{\beta_L^2}{\left(1-\beta_L\right)^2}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right) \\
&+ \frac{\beta_L^3}{\left(1-\beta_L\right)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{\beta_L}{1-\beta_L}\log(1-\beta_L)
\end{aligned}$$

s.t.

$$(1-\phi_{T-1})(1+r_{\ell,T-1})L_{T-1} \geq (1+r_{T-1})B_{T-1}.$$

The decision rules of this problem are given as

$$\begin{aligned}
L_{T-1} &= \frac{\beta_L(1+r_{T-1})}{1+r_{T-1}-(1-\phi)(1+r_{\ell,T-1})}\omega_{t-2} \\
B_{T-1} &= \frac{\beta_L(1-\phi_{T-1})(1+r_{\ell,T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\omega_{t-2} \\
L_{T-1}-B_{T-1} &= \beta_L\omega_{t-2} \\
(1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1} &= \frac{\beta_L\phi_{T-1}(1+r_{\ell,T-1})(1+r_{T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\omega_{t-2}, \\
\omega_{t-2} &= \left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}\right),
\end{aligned}$$

which give

$$\begin{aligned}
\Psi_{T-2}\left(L_{T-2},B_{T-2}\right) &= \frac{1}{1-\beta_L}\log\left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_{T-1}(1+r_{\ell,T-1})(1+r_{T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\right) \\
&+ \frac{\beta_L^2}{1-\beta_L}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right) \\
&+ \frac{\beta_L^3}{\left(1-\beta_L\right)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).
\end{aligned}$$

The value function when the bank defaults is

$$\begin{aligned}\tilde{\Psi}^{D}_{T-2}\left(\xi(1+r_{\ell,T-2})L_{T-2}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T-2})L_{T-2}\right) + \frac{\log(1-\beta_L)}{1-\beta_L} \\ &+ \frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r_{T-1})) + \frac{\beta_L^2}{1-\beta_L}\log(\beta_L(1+r_T)) \\ &+ \frac{\beta_L^3}{(1-\beta_L)^2}\log(\beta_L(1+r)).\end{aligned}$$

The no-default condition in period $T-2$ can be written as

$$(1-\phi_{T-2})(1+r_{\ell,T-2})L_{T-2} \geq (1+r_{T-2})B_{T-2},$$

where

$$\phi_{T-2} = \xi^{1-\beta_L}\left(\frac{1+r_{T-1}}{1+r_{\ell,T-1}} - (1-\phi_{T-1})\right)^{\beta_L}.$$

The derivations suggest that the value functions and decision rules have the same pattern. Thus, they will take the same form of the previous period. □

F.3 Bank's solution

Given the collateral constraint the bank is facing, we can explicitly solve for the bank's problem, which is summarized in the following proposition.

Proposition 3. *The decision rules when the no-default constraint binds (if $r_{\ell,t+1} > r_{t+1}$) are*

$$\begin{aligned}L_{t+1} &= \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\beta_L\omega_t \\ B_{t+1} &= \frac{(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\beta_L\omega_t,\end{aligned}$$

where $\omega_t = (1+r_{\ell,t})L_t - (1+r_t)B_t$.

The decision rules when the no-default constraint does not bind (if $r_{\ell,t+1} \leq r_{t+1}$) are:

$$B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\omega_t\right] & \textit{if } r_{\ell,t+1} = r_{t+1} \\ 0 & \textit{if } r_{\ell,t+1} < r_{t+1} \end{cases}$$

and

$$L_{t+1} = B_{t+1} + \beta_L \left((1 + r_t^*)L_t - (1 + r_t)B_t\right).$$

F.4 Characterization of the Bank's Problem in Stationary Equilibrium

We can further characterize the bank's problem under stationarity. Throughout the paper, we will focus on stationary equilibria where the capital requirement constraint is binding. If it did not bind, then bank balance sheets would not have any impact on the economy. However, we do not rule out the case that there might be some periods in the transition where this constraint becomes slack. Using the general formula capturing both the exogenous and endogenous capital requirement constraint, we have the following decision rules when the constraint binds:

$$L_{t+1} = \beta_L \widehat{\lambda}_t \omega_t \quad \text{and} \quad B_{t+1} = \beta_L \left(\widehat{\lambda}_t - 1\right) \omega_t,$$

where

$$\widehat{\lambda}_t = \frac{(1 + r_{t+1})}{1 + r_{t+1} - (1 - \phi_{t+1})(1 + r_{\ell,t+1})}. \tag{11}$$

Then the law of motion for net worth is given as

$$\omega_{t+1} = L_{t+1}\left(1 + r_{\ell,t+1}\right) - B_{t+1}\left(1 + r_{t+1}\right).$$

Then, we can obtain the next period's net worth as

$$\omega_{t+1} = \beta_L \left(\widehat{\lambda}_t \left(1 + r_{\ell,t+1}\right) - \left(\widehat{\lambda}_t - 1\right)\left(1 + r_{t+1}\right)\right) \omega_t.$$

Imposing steady state $\omega_{t+1} = \omega_t$ and $\widehat{\lambda}_t = \widehat{\lambda}$ gives

$$r_\ell - r = \frac{1 - \beta_L(1+r)}{\widehat{\lambda}\beta_L},$$

where $r_\ell - r$ is the premium due to the bank capital constraint. If $\beta_L(1+r) < 1$ and $\widehat{\lambda} < \infty$, then $r_\ell - r > 0$. Thus, the capital constraint will be binding in the stationary equilibrium. To understand this point, assume that $\beta_L(1+r) < 1$ but the bank starts with a high net worth so that the capital requirement constraint is not binding. In that case, $r_{\ell,t+1} = r$ and the bank's decision rule is $L_{t+1} - B_{t+1} = \beta_L \omega_t$. Using that, we can show that $\omega_{t+1} = (1+r)\beta_L \omega_t < \omega_t$. Thus, the bank consumes from its net worth until the capital constraint starts to bind. Thus, the economy will converge to a stationary equilibrium where it actually binds.

G Computational Algorithm

Denote the state variable of the household as $\theta = (a, h, d, z, j, i, s)$ where s is the housing tenure, i is the indicator whether the individual is a depositor or a capitalist, j is the age of the household, z is the income efficiency shock, d is the ratio of mortgage debt to initial house price level, h is the size of owner-occupied unit, and a is the financial wealth after the return is realized. For active/inactive renters ($s \in \{r, i\}$) $h = d = 0$. We discretize a into 120 and d into 60 exponentially spaced points. The age j runs from 1 to 30 and h is linearly discretized into 5 points. Income shock z is discretized into 15 points, and grid points and transition probabilities are computed using Tauchen's method. Since this is a life-cycle model, the grid points for income shocks are age dependent to better approximate the AR(1) process with a Markov process.

G.1 Steady-State Computation

The steady state of the model is computed as follows:

1. From the bank's problem, the lending rate at the steady state is $r^* = r + \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$.

2. Make a guess on K and p_h.

3. Given these guesses, using the firm's problem, compute w and u:

$$\begin{aligned} u &= \left(\frac{(1-\alpha)K}{(1+\phi r_\ell)\vartheta}\right)^{\frac{1}{\alpha+\psi}} \\ w &= \vartheta^{\frac{\alpha-1}{\alpha+\psi}}\left(\frac{(1-\alpha)K^\alpha}{(1+\phi r_\ell)}\right)^{\frac{1+\psi}{\alpha+\psi}} \\ \tilde{r} &= \alpha\left(\frac{K}{u}\right)^{\alpha-1} - \delta \end{aligned}$$

4. Using the rental companies' problem, compute the rent price:

$$p_r = \kappa + \frac{1-\delta_h}{1+\tilde{r}} p_h$$

5. Given all these prices, solve the household's problem recursively:

   (a) Solve the terminal period problem where all dynamic choices are set to 0: $a' = d' = 0$. This gives the value for the household, $V_J(\theta)$, and the continuation value of the mortgage contract, $v_J^l(\theta)$.

   (b) Given $V_j(\theta)$ and $v_j^l(\theta)$, solve $V_{j-1}(\theta)$ and $v_{j-1}^l(\theta)$:

      i. Given $V_j(\theta)$ and $v_j^l(\theta)$, first solve the expected continuation values $EV_j(\theta)$ and $Ev_j^l(\theta)$.

      ii. Solve for mortgage prices at the origination, $q^m(\theta)$, using equations 2 and **??**.

      iii. The solutions of the problems for the inactive renter and the active renter who decides to become a renter are straightforward. Their choices are housing services, consumption, and saving. We interpolate the expected value of the continuation value using linear interpolation, and to choose the optimal saving level, we first search globally over a finer discrete space for a' to bracket the maximum.[46] Once the maximum is bracketed, we solve for the optimum using Brent's method. Given the saving choice, we compute the optimal housing

[46]For saving choice, we use 240 grid points.

services using the analytical expression for it.[47] Then, we use the budget constraint to compute the consumption.

iv. The most complex and time-consuming problem is the problem of the renter who decides to purchase a house. This household chooses consumption, saving, house size, and mortgage debt. We restrict the choice of down payment and house size to finite sets. For down payment, the grid points for d are the choices, and for house size the grid points for h are the choices.[48] For each down payment and house size choices, we solve household's objective function, $V_{j-1}^{d,h}$, by finding the optimal saving level as we discussed in point 5(b)iii. Given all household choices, we can obtain $q^m(\theta)$. We use linear interpolation for the points off the grid. Also given the choice of d and h, the mortgage debt becomes $dp_h^* h$ where p_h^* is the equilibrium price level at the initial steady state. Once the objective function is solved for a given down payment and house size choice, we set $V_{j-1}(\theta) = max_{d,h}\left\{V_{j-1}^{d,h}\right\}$.

v. The solution of the homeowner's problem:

   A. Stayer: The stayer's problem is simple since the household only chooses consumption and saving. It is solved similar to the inactive renter's problem. The only exception is that in the continuation value, the variable keeping track of the principal amount d will be adjusted. Given current d, $d' = (d - m)(1 + r^*)$ where $m = \frac{r^*(1+r^*)^{J-j}}{(1+r^*)^{J-j+1}-1}$. We use linear interpolation over d' to compute the expected continuation value for the household.

   B. Seller: The seller's problem is the same as the problem of an active renter except for the fact that in the budget constraint, the household will have the term due to the proceedings from the sale of the house: $p_h h(1-\varphi_s) - dhp_h^*$

[47] Since utility is Cobb-Douglas in non-durable consumption and housing services, we can obtain an analytical expression for optimal housing services.

[48] Increasing the number of grid points for d and h beyond the levels we set does not noticeablyc hange the results.

C. Refinancer: The refinancer's problem is the same as the problem of a renter who purchases a house except for the fact that she is restricted to purchasing the same house.

D. Defaulter: The defaulter's problem is the same as the active renter's problem.

vi. Solving the homeowner's problem also gives us the mortgage payments for each type of mortgage contract and allows us to compute the continuation of the mortgage contract, $v_j^l(\theta)$:

$$v_{j-1}^l(\theta) = m(\theta) + \frac{1}{1+r^*}\int_{\theta'} v_j^l(\theta')\,\Pi(\theta'|\theta),$$

where

$$m(\theta) = \begin{cases} dhp_h^* & \text{if } s \in \{hr, hf\} \\ p_h h(1-\varphi_e) & \text{if } s = he \\ \frac{r^*(1+r^*)^{J-j}}{(1+r^*)^{J-j+1}-1} dhp_h^* & \text{if } s = hh \end{cases}$$

(c) Repeat step (b) for each $j = \{J-1, ..., 1\}$.

6. Given the policy functions for the household, simulate the economy $N = 20,000$ individuals for $J = 30$ periods. This gives us aggregate saving, A, aggregate housing demand, H^d, and aggregate rental demand, H^r. Given aggregate saving, we update the aggregate capital guess as $K = (1-\lambda_k)K + \lambda_k(A - V^{rc}(H^r))$ where $V^{rc} = \frac{p_r - \kappa - \delta_h p_h^0}{\tilde{r}} H^r$ is the value of rental companies. Given aggregate housing demand, we update the house price guess as $p_h = p_h\left(1 + \lambda_h \frac{H-\bar{H}}{\bar{H}}\right)$. We set $\lambda_k = \lambda_h = 0.1$. We continue this process until $max\left(|A - W(H^r) - K|, \left|H - \bar{H}\right|\right) < \epsilon$ where $\epsilon = 10^{-4}$.

7. Once equilibrium prices and allocations are solved, we solve for bank-related variables: bank net worth, bank assets, and bank liabilities using the steady-state analytical equations for these variables.

G.2 Transition Algorithm

The transitional problem has two main differences. First, we need to solve for a path of equilibrium prices and allocations along the transition. Second, we need to adjust the algorithm to capture the fact that the risk-free mortgage interest rate can change along the transition. This second point is important because in order to save from state variables, we assume individuals pay points at the origination time to reduce the risk-adjusted mortgage interest rate to the risk-free mortgage interest rate. This allows us to get rid of the mortgage interest rate as an additional state variable. However, since shocks are permanent, this assumption can artificially distort the equilibrium. Consider a decline in the risk-free mortgage interest rate from 5 percent to 4 percent. If we still assume all new mortgages are priced at 5 percent, this would imply that banks would be paid more than the principal amount if they still use the same amortization schedule we use in the steady-state algorithm. That will result inq^m being significantly larger than 1, implying a substantial subsidy from banks to individuals. More importantly, if we also apply this new risk-free mortgage interest rate to existing mortgages, that would imply a reduction of all the existing mortgage payments: a positive wealth shock to all existing mortgage owners and a negative shock to banks.[49]

To tackle this issue without further complicating the solution algorithm, we assume that after the shock is realized, all new mortgages will be priced at the new risk-free mortgage rate, whereas all existing mortgages will be still paid using the old risk-free mortgage rate. We also include an additional state variable to the household's problem to keep track of whether the household purchased a house before or after the shock is realized. This allows us to compute the mortgage payments more accurately without substantially distorting the solution algorithm.

Given these modifications, the rest of the algorithm is as follows:

1. Fix the time it takes for the transition to happen: T periods. We set $T = 60$ corre-

[49]Since we keep track of the principal balance as a state variable, we need to know the risk-free mortgage rate to compute the implied mortgage payments. Another formulation could be to keep track of the mortgage payments. However, in this case, we still need to know the risk-free mortgage rate in order to compute the implied principal amount since it affects the resources of homeowners in the event of selling/refinancing/defaulting.

sponding to 120 years.

2. Solve the initial steady state of the problem as outlined above. Store the initial steady-state distribution denoted as $\Gamma_0(\theta)$.

3. Given the boom shock, solve the final steady state of the problem as outlined above. Store $V_T(\theta)$ and $v_T^l(\theta)$.

4. Guess the path of aggregate capital stock, rental demand, house price and lending rate: $\left\{K_{t+1}, H_t^{r,0}, p_t^h, r_{t+1}^{*,0}\right\}_{t=1}^{T-1}$

5. Given these guesses, compute $\{w_t, \tilde{r}_{t+1}, p_t^r\}$ using the good-producing firm's and rental companies' problem. Compute V_t^{rc} using the rental companies' problem.

6. Solve each cohort's problem for each period they are alive, starting from the cohort born in period $-J+2$ until the cohort born in period $T-1$[50]:

    (a) For each generation, given prices, solve the household's problem and the continuation value of the contract as in the steady-state problem above. The only difference is that for new mortgage buyers, the risk-free mortgage interest rate is the final steady-state risk-free mortgage interest rate, whereas for existing mortgage owners, it is the initial steady-state risk-free mortgage interest rate. This also affects the continuation value for households and mortgage contracts since we need to keep track of whether a mortgage originated before or after the shock.

    (b) Given the policy functions for each generation, simulate the economy starting from the initial steady-state distribution $\Gamma_0(\theta)$ for T periods. We fix the same random numbers for the idiosyncratic shocks to household.

    (c) Using the simulated path, compute the aggregates: $A_{t+1}, H_t^{r,1}, H_t^d$, $M_t = \int v_t^l(\theta)$.

[50] A household of age j belonging to a cohort born in period $g \in \{-J+2, ..., T-1\}$ will be subject to prices p_{g+j-1}.

(d) Update guesses:

$$
\begin{aligned}
K_{t+1} &= (1-\lambda_k)\, K_{t+1} + \lambda_k \left(A_{t+1} - V^{rc}_{t+1}\left(H^r_{t+1}\right)\right) \\
H^r_t &= (1-\lambda_{rc})\, H^{r,0}_t + \lambda_{rc} H^{r,1}_t \\
p^h_t &= p^h_t \left(1 + \lambda_h \frac{H^d_t - \bar{H}}{\bar{H}}\right) \\
r^{*,0}_{t+1} &= (1-\lambda_r)\, r^{*,0}_{t+1} + \lambda_{rc} r^{*,1}_{t+1}
\end{aligned}
$$

where r^*_{t+1} solves

$$
L_{t+1} = \frac{(1+r_{t+1})}{1 + r_{t+1} - (1-\phi_{t+1})(1+r^*_{t+1})} \beta_L N_t
$$

where $L_{t+1} = M_{t+1} + \phi w_{t+1}\left(\bar{w}, u_{t+1}\right)$ and

$$
N_t = \begin{cases} L_t\left(1+r^*_t\right) - B_t\left(1+r_t\right) & \text{if } t = 1 \\ \left(1+r^{*,0}_t\right)\phi_t L_t & \text{if } t > 1 \end{cases}
$$

(e) Iterate this process until convergence occurs on guesses. The convergence criteria are defined as $\max\left|K_{t+1} + V^{rc}_{t+1}\left(H^r_{t+1}\right) - A_{t+1}\right| < \epsilon_k$, $\max\left|H^{r,1}_t - H^{r,0}_t\right| < \epsilon_h$, $\max\left|H^d_t - \bar{H}\right| < \epsilon_h$, and $\max\left|r^{*,1}_t - r^{*,0}_t\right| < \epsilon_r$ where $\epsilon_k = \epsilon_h = 10^{-3}$ and $\epsilon_r = 10^{-4}$.