Credit Supply, Housing Demand, and Bank Balance Sheets in the Great Recession*

Yavuz Arslan Bulent Guler Burhan Kuruscu

Abstract

We quantify the relative contributions of credit supply and housing demand shocks to the dynamics of the US housing market and macroeconomy around 2008. Using a general equilibrium model with households, firms, and banks, we find that credit supply changes, arising from both exogenous shocks to bank leverage and endogenous adjustments in bank balance sheets, played a significant role in the boom-bust cycle. While the housing demand shock substantially contributed to the house price boom, its impact was amplified during the bust as it led to increased foreclosures, deteriorating bank balance sheets, and consequently, reduced credit supply.

JEL Codes: E21, E32, E44, G21, G51.
Keywords: Credit Supply; House Prices; Financial Crises; Household, Firm, and Bank Balance Sheets; Leverage; Foreclosures, Mortgages; Consumption; and Output.

*We thank Pedro Gete, Gaston Navarro, and Dominik Supera for their valuable discussions of our paper. We thank Stijn Claessens, Dean Corbae, Victor Rios-Rull, Hyun Shin, Gianluca Violante and participants at various conferences. Kuruscu thanks the Social Sciences and Humanities Research Council of Canada. The authors also acknowledge the Indiana University Pervasive Technology Institute for providing computing resources that have contributed to the research results reported within this paper. Arslan: University of Liverpool Management School, yavuz.arslan@liverpool.ac.uk; Guler: Indiana University, bguler@indiana.edu; Kuruscu: University of Toronto, burhan.kuruscu@utoronto.ca.

1 Introduction

In this paper, we study the underlying factors behind the boom-bust cycle that occurred in the US housing market circa 2008.[1] For this purpose, we develop a general equilibrium model that combines three sectors of the economy: (i) an overlapping-generations structure of households who face idiosyncratic income risk under incomplete markets, make housing tenure decisions, and borrow through long-term defaultable mortgages; (ii) banks that issue short-term loans to firms and long-term mortgages to households and whose ability to intermediate funds depends on their capital due to a limited enforcement friction; and (iii) firms that finance part of their wage bill (working capital) through short-term loans from banks. We use the model to quantify (1) the importance of credit supply versus housing demand shocks for the boom-bust cycle as observed in the US around 2008 and (2) how much the feedback from bank balance sheets to households and firms amplified the Great Recession.

To study the boom-bust episode, we calibrate the model to match several US data moments regarding household and bank balance sheets before 1998. We then introduce two unexpected shock pairs: leverage shocks to bank balance sheets and housing demand shocks. In 1998, banks began increasing their leverage, expanding credit supply. The size of this shock is calibrated to match the observed changes in banks' book leverage during the boom, accounting for 35 percent of the house price increase from 1998 to 2006. We then calibrate a housing demand shock, following Kaplan et al. (2020), to account for the remaining portion of the house price boom. In 2008, both shocks are reverted to their initial steady-state values, triggering the financial crisis.[2]

The benchmark economy features a boom-bust cycle similar to the one observed in the US: house prices, GDP, consumption, bank loans, mortgage debt, and bank leverage rise significantly during the boom and contract sharply

[1] See Gertler and Gilchrist (2018) for a review of the crisis and the literature.

[2] Additionally, we introduce government bailouts to banks and households, calibrating their sizes to match the decline in bank net worth and the increase in household foreclosures, respectively. Our analysis suggests that the banking sector would have collapsed in the absence of these bailouts.

during the bust, and the credit spread spikes at the time of the bust, consistent with the data. The model's cross-sectional implications are also consistent with the recent evidence from detailed micro-level data analysis. In particular, credit grows similarly across different income quantiles during the boom, as in the data (Adelino et al. (2016), Foote et al. (2016), and Albanesi et al. (2017)).

Equipped with a model that can generate economic dynamics that are close to the data, we study the drivers of the boom-bust cycle. However, before we proceed, it will be instructive to discuss the transmission mechanism of each shock. First, exogenous shocks to bank leverage translate into changes in bank credit supply, which in turn affect the equilibrium bank lending rate. Changes in the bank lending rate affect model dynamics both *directly* via household borrowing costs and *indirectly* through their effect on labor income (since firms partly depend on bank borrowing). The housing demand shock, on the other hand, affects household wealth and the collateral constraint via its effect on house prices. During the bust, however, another effect of the housing demand shock also becomes important. Because of the large decline in house prices, foreclosure rates increase significantly, which lowers bank net worth and hence credit supply. With a lower credit supply, the bank lending rate rises, leading to higher costs of borrowing for both households and firms. Firms reduce labor demand, and equilibrium wages decline. The same endogenous credit supply channel also amplifies the effect of the bank leverage shock during the bust.

Our results show that the bank leverage shock accounts for almost all of the decline in the bank lending rate, the rise in output and consumption during the boom, and generates a substantial rise in house prices (more than one-third of the benchmark increase) in the model. The housing demand shock, on the other hand, accounts for a larger increase in house prices, a small increase in consumption, but no increase in output during the boom. Turning to the bust, we find that both shocks have large effects. Compared to the boom, both shocks affect all aggregates more during the bust because of the amplification generated by the bank balance sheet deterioration during that time. However, this amplification is more prevalent for the housing demand shock.

Through a series of experiments, we show that the feedback from the bank balance sheets to household and firm borrowing rates (the latter affecting household labor income) amplifies the effects of shocks. First, we analyze the effects of the changes in the equilibrium bank lending rate of our benchmark economy, which is driven by the changes in credit supply. The changes in the bank lending rate have a permanent component due to permanent changes in bank leverage in the boom-bust cycle but also a temporary spike due to the deterioration of the bank balance sheet at the time of the bust. We find that the decline in the bank lending rate during the boom accounts for one-third of the increase in house prices and three-quarters of the increase in consumption. We then explore how the changes in the bank lending rate affect consumption and house prices *directly* through household borrowing costs and *indirectly* through firm borrowing costs and hence wages. Our analysis reveals that the direct and indirect effects are comparable for house prices; however, the indirect effect is far more important for consumption since the change in consumption is driven by changes in both house prices and wages. During the bust, we find that the bank lending rate has larger effects, generating one-half of the house price and 80 percent of the consumption declines. As in the boom, its direct and indirect effects are comparable for house prices, and its indirect effect is more important for consumption.

Second, we quantify the importance of the bank balance sheet deterioration—which manifests its effect through the spike in the bank lending rate during the bust—and find that it accounts for about 90 percent of the labor income, one-third of the house price, and more than one-half of the consumption benchmark declines. These results imply that the bank balance sheet deterioration amplifies the bust in variables that depend on short-term debt, such as labor income, but it has a relatively smaller effect on house prices, which depend on long-term debt, and a somewhat intermediate effect on consumption, which is driven by both house prices and labor income.[3] When we decompose the effect of the spike into its direct and indirect effects, we find similar effects

[3] Gertler and Gilchrist (2018) provide evidence that the disruption in banking, as in our model, was central to the overall employment contraction in the data.

on house prices; however, the indirect effect is more important for the decline in consumption, as in the previous experiment.

Third, we focus on the effect of the housing demand shock during the bust. We find that the endogenous decline in credit supply accounts for one-third of the overall effect of the housing demand shock on house prices and more than half of its effect on consumption during the bust.[4] As in the previous exercises, we further quantify the direct and indirect effects of this credit supply channel. While its direct and indirect effects are both important for house prices, its indirect effect turns out to be more important for consumption, a recurring theme that is present in all three experiments. Finally, we find that the credit supply response to the housing demand shock increases nonlinearly with the size of the shock. This is because households hold some equity in their houses and small declines in house prices do not increase foreclosures significantly enough to hurt bank balance sheets.

Related literature

Our paper contributes to the literature that studies the dynamics of the housing market and the macroeconomy around the 2008 financial crisis. Using a model of representative borrower and saver, Justiniano et al. (2017) demonstrate that credit supply, driven by looser lending constraints in the mortgage market, accounts for the unprecedented rise in home prices, the surge in household debt, the stability of debt relative to home values, and the fall in mortgage rates.[5] However, Kaplan et al. (2020) argue that the absence of the rental market and/or long-term defaultable mortgages are critical for obtaining large effects of credit supply or credit conditions on house prices since, with rental markets, households can rent a house of their desired size if they are constrained in purchasing one. With these extensions, Kaplan et al. (2020) argue that shifts

[4]During the boom, the credit supply response to the housing demand shock is negligible.

[5]In a similar vein, Kiyotaki et al. (2011) and Adam et al. (2012) find that the decline in interest rates contributed substantially to the house price boom in the U.S. On the other hand, Greenwald (2016), using representative borrower and savers, and Huo and Rios-Rull (2013), Sommer et al. (2013), and Favilukis et al. (2017), using heterogeneous agent frameworks, show that changes in maximum LTV or payment-to-income (PTI) ratios can generate significant changes in house prices and consumption.

in household demand due to shocks to house price expectations, rather than changes in credit supply or conditions, were the main driving force behind the boom-bust cycle in the housing market. They also find that temporary shocks to interest rate, essentially a credit supply shock, does not move house prices.

Despite modeling detailed household structure, similar to Kaplan et al. (2020), we find a more significant role for credit supply due to two key differences.[6] First, we consider permanent changes in bank leverage (and hence the bank lending rate) rather than the LTV, PTI, or temporary interest rate shocks. Second, the credit supply shock in our framework is not an isolated shock to households since firms also need to borrow from banks to produce output. Thus, the changes in credit supply—due to exogenous shocks to bank leverage and/or endogenous changes in bank balance sheets—generate changes in the bank lending rate, which then affect households both directly through their borrowing cost and indirectly through firm borrowing costs. This, in turn, creates a boom-bust cycle in the housing market and the rest of the macroeconomy. Moreover, our analysis highlights the importance of endogenous credit contractions due to deteriorating bank balance sheets in amplifying the crisis.

Landvoigt (2016), and Diamond and Landvoigt (2022) also combine banking and household sectors, as in our model.[7] Our paper has many points of contact with Diamond and Landvoigt (2022), who also show the importance of credit supply for the boom-bust in house prices and mortgage debt. Different from Landvoigt (2016) and Diamond and Landvoigt (2022), we model the feedback from banks' credit supply to firm borrowing, which significantly contributes to the boom-bust. Furthermore, the richer heterogeneity in our household sector allows us to compare our model's implications with cross-sectional facts, which were argued to be against the credit supply channel.

Mechanisms in our model are supported by empirical findings as well. First,

[6]Nevertheless, we confirm their findings that LTV, PTI, or temporary interest rate shocks barely move house prices.

[7]Elenev (2017), Elenev et al. (2016), and Elenev et al. (2018) also use an approach similar to these papers to address different questions from ours.

with detailed data from periods after 1996, Fraisse et al. (2020), Gropp et al. (2019), Aiyar et al. (2014), De Marco et al. (2021), Jiménez et al. (2017), and Gete and Reher (2018), causally link regulatory tightenings to declines in credit, and contractions in economic activity. Second, Gilchrist and Zakrajšek (2012) and Gertler and Gilchrist (2018) show that credit spreads spike during downturns, predicting declines in subsequent economic activity. The credit spread dynamics in our model are similar to the excess bond premium dynamics reported in these papers. Third, Glaeser et al. (2012) and Justiniano et al. (2017) find that interest rates on firm loans and mortgages declined during the boom. Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, Ivashina and Scharfstein (2010) document a more than 50 percent decline in capital expenditure and working capital loans to corporations. Similarly, Adrian et al. (2013) find that capital expenditure and working capital loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis.

Finally, our framework combines key elements from two strands of literature. On the one hand, an active literature studies the pricing of default risk in the context of household debt, but abstracting from the bank balance sheet effects.[8] On the other hand, the literature on bank balance sheets has studied how depletion of banks' capital reduces their ability to intermediate funds.[9] However, in this literature, banks' asset structure typically takes a simple form, such as one-period bonds, or lacks the rich heterogeneity observed in banks' portfolios. By combining these two strands of the literature, our model allows us to study the rich interactions among households, firms, and banks.

[8] Among others see Chatterjee et al. (2007), Livshits et al. (2010, 2007), Jeske et al. (2013), Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), Arslan et al. (2015), Guler (2015), Hatchondo et al. (2015), Mitman (2016), Gete and Zecchetto (2024), and Kaplan et al. (2020).

[9] See Mendoza and Quadrini (2010), Gertler and Kiyotaki (2010, 2015), Gertler and Karadi (2011), Bianchi and Bigio (2014), and Corbae and D'Erasmo (2013, 2019). See also Bernanke and Gertler (1989), Bernanke et al. (1999), Kiyotaki and Moore (1997), and Buera and Moll (2015) which have studied the financial accelerator mechanism in the context of non-financial firms.

2 Quantitative Model

The model economy is composed of five sectors: (i) households, (ii) financial intermediaries (banks), (iii) rental companies, (iv) firms, and (v) the government.

Total housing stock is fixed at $\bar{H}$, but the homeownership rate is not. This becomes possible as part of the housing stock is owned by homeowners and the rest is owned by rental companies who rent it to the households. There is perfect competition in all markets.

There is no aggregate uncertainty. Boom-bust transitions are generated by two unexpected shocks, both perceived as permanent. Other than the shock periods, there is perfect foresight. Since households are ex post heterogeneous, all the endogenous prices, value functions, and policy functions depend on the aggregate state of the economy and the distribution of households. For notational convenience, we suppress these dependencies.

2.1 Households

We assume that households supply labor inelastically until the mandatory retirement age J_r and live until age J (with $J > J_r$). A household's income process $y(j, z_j)$ is given by $y(j, z_j) = (1 - \tau)\, w \exp(f(j) + z_j)$ for $j \leqslant J_r$ and $y(j, z_j) = w y_R(z_{J_r})$ for $j > J_r$, where $f(j)$ captures the life-cycle component of income and $z_j = \rho z_{j-1} + \varepsilon_j$ with $\varepsilon_j \sim \text{i.i.d.}\ N(0, \sigma_\varepsilon^2)$. The variable w is the wage per efficiency units of labor, and τ is the labor income tax rate. Function $y_R(z_{J_r})$ approximates the US retirement system.

Households receive utility from consumption and housing services and can choose between renting and owning a house of their desired size. Household preferences take the following form: $E_0[\sum_{j=1}^{J} \beta^{j-1} u(c_j, s_j)]$, where E_0 is the expectations operator, β is the discount factor, c_j is consumption, and s_j is housing services at age j.

Housing Choices: Households enter the economy as *active* renters and can stay as renters by renting a house at the desired size at the price p_r per unit. They can also purchase a house and become homeowners at any time. There is no unsecured borrowing in the model. However, households have access to the mortgage market to finance their housing purchases subject to a minimum down payment requirement. The terms of mortgage contracts, down payment, and mortgage pricing are endogenous and depend on household characteristics. Homeowners can choose to stay as homeowners or become renters again, by either selling their houses or defaulting on mortgages. Homeowners can pay the existing mortgage or obtain a new one through refinancing. Households also can upgrade or downgrade their houses by selling the current house and buying a new one.[10]

Several transaction costs are associated with housing market transactions. A seller has to pay φ_s fraction of the selling price. Obtaining a mortgage from banks involves a fixed cost (φ_f) and a variable cost (φ_m) as a fraction of the mortgage debt at the origination.

Defaulting on a mortgage is possible but costly. After default, households become *inactive* renters; that is, they temporarily lose access to the housing market. Inactive renters become active renters with probability π. Therefore, agents have three statuses regarding their housing decision: homeowner, active renter, or inactive renter.

Mortgage Payments: For tractability, we assume that mortgages are due by the end of life, so that the household's age captures the maturity of the mortgage contract. We also allow for only fixed rate mortgages. Therefore, the mortgage contract can be characterized by its maturity and the periodic mortgage payment m. We assume that the mortgage payments follow the standard amortization formula computed at the bank lending rate r_ℓ.[11]

[10]The household sector builds on the ones in Arslan et al. (2015) and Guler (2015) but is extended in important ways, such as refinancing, flexible housing and rental sizes..

[11]The mortgage interest rate differs across households since ex post households are heterogeneous. Ideally, the amortization schedule should be computed at the individual mortgage interest rate instead of r_ℓ. However, to avoid using an additional state variable,

Optimization Problem of Households: We present the optimization problem of a purchaser here (the rest of the optimization problems are in Appendix E). If an active renter chooses to purchase a house, she chooses the mortgage debt level d that determines $q^m(d; a', h, z, j)$, the price of the mortgage at the origination, which is a function of the current state of the household (income realization z and age j), house size h, and asset choice a'. Then, the optimization problem of an active renter who chooses to buy a house is given by

$$V_j^{rh}(a, z) = \max_{c,d,h,a' \geqslant 0} \left\{ u(c, h) + \beta E V_{j+1}^{h}(a', h, d, z') \right\} \tag{1}$$

subject to

$$\begin{aligned} c + (1 + \delta_h) p_h h + \varphi_f + \frac{a'}{1 + r} &= y(j, z) + a + d \left(q^m(d; a', h, z, j) - \varphi_m \right), \\ d &\leqslant (1 - \iota) p_h h, \end{aligned}$$

where V^h is the continuation value for a homeowner, p_h is the house price, δ_h is the proportional maintenance cost of housing, and ι is the down payment requirement.

2.2 Firms

A continuum of perfectly competitive firms produce output by combining capital K_t and labor N_t. The firm also chooses hours per worker (or worker utilization rate), u_t. The wage per efficiency units of a worker $w(\overline{w}_t, u_t)$ (same as w in $y(j, z_j)$) is assumed to depend on the hours worked, that is, $w(\overline{w}_t, u_t) = \bar{w}_t + \vartheta \frac{u_t^{1+\psi}}{1+\psi}$, where ϑ and ψ are constants. In this formulation, hours are chosen by the firm, and workers are assumed to supply hours at no cost, but u_t, $\overline{w}_t$, and hence $w(\overline{w}_t, u_t)$ are determined in equilibrium. This formulation generates a positive relation between aggregate hours and wages that mimics an aggregate labor supply response to aggregate wages.

we assume that mortgage amortization is computed at r_ℓ, as in Hatchondo et al. (2015) and Kaplan et al. (2020). Then, individual default risk will show up in the pricing of the mortgages at the origination rather than in the mortgage interest rate. Thus, essentially all households pay a premium at the origination to reduce the mortgage interest rate to r_ℓ.

The firm finances a fraction μ of the wage payment in advance from banks and pays interest on that portion. Then, the firm's problem is given by

$$\max_{K_t, N_t, u_t} \mathbb{Z}_t K_t^{\alpha} \left(N_t u_t\right)^{1-\alpha} - (r_{k,t} + \delta_k) K_t - (1 + \mu r_{\ell,t+1})\, w\left(\overline{w}_t, u_t\right) N_t,$$

where $\mathbb{Z}_t$ is TFP, $r_{k,t}$ is the rate of return, and δ_k is the depreciation rate of capital. Since a worker's labor income depends on hours worked, labor income and output decline when the firm reduces work hours in response to an increase in bank lending rate r_ℓ.[12]

2.3 Rental Companies

A rental company enters period t with $(1 - \delta_h) H^r_{t-1}$ units of rental housing stock where δ_h is the depreciation rate of housing. Then, it chooses H^r_t. In that period, the company receives net rent $(p^r_t - \kappa) H^r_t$ and pays dividend $x^r_t = p^h_t (1 - \delta) H^r_{t-1} - p^h_t H^r_t - \frac{\eta}{2} p^h_t \left(H^r_t - H^r_{t-1}\right)^2 + (p^r_t - \kappa) H^r_t$ to shareholders where p^r_t is the rental price per unit of housing and κ is the maintenance cost. The expression $\frac{\eta}{2} p^h_t \left(H^r_t - H^r_{t-1}\right)^2$ is the quadratic adjustment cost of changing rental supply. A higher η implies a more segmented housing market.

Since both capital and rental company shares are riskless in the steady state and along the transition path, both assets pay the same return except for the two unanticipated shock periods.[13] Given this, the first-order condition of the rental company gives the rental price as a function of house price and rental housing stock in periods $t-1$, t, and $t+1$: $p^r_t = \kappa + p^h_t + \eta p^h_t \left(H^r_t - H^r_{t-1}\right) - \frac{(1-\delta_h) p^h_{t+1} + \eta p^h_{t+1}\left(H^r_{t+1} - H^r_t\right)}{1 + r_{k,t+1}}$. This formulates the supply for the rental housing. The demand for rental housing comes from households' housing choices.

[12] We could have achieved the same effect with endogenous labor supply. In that case, the firm would reduce labor demand, which would reduce wages. Households would reduce labor supply and output would decline. We choose this formulation because it is easier to handle computationally.

[13] At the time of an unexpected shock, capital and the rental housing return could be different. Then, the realized return will be different from the contracted return, and these profits/losses are borne by the households proportional to their asset holdings.

2.4 Banks

We assume a competitive banking industry with a continuum of identical banks that are risk-averse and maximize $\sum_{t=0}^{\infty} \beta_L^{t-1} \log\left(c_t^B\right)$, where c_t^B is the banker's consumption. There is no entry or exit from the banking sector. Banks fund their operations from their net worth ω_t and by borrowing B_{t+1} in the international market at a risk-free interest rate r_{t+1}. They lend L_{t+1}^k to the firm at $r_{\ell,t+1}$, and issue mortgages and purchase existing mortgages.

Let $\theta = (d; a, h, z, j)$ define the type of a mortgage, ω_t be the bank's net worth, and $\ell_{t+1}(\theta)$ be the amount of investment in mortgage type θ (which includes any newly issued as well as existing mortgages). The budget constraint of the bank is given by

$$c_t^B + L_{t+1}^k + \int_\theta p_t(\theta)\, \ell_{t+1}(\theta) = \omega_t + B_{t+1},$$

where $p_t(\theta)$ is the price of a type-θ mortgage after the mortgage payment $m_t(\theta)$. The bank's net worth evolves according to

$$\omega_{t+1} = \int_\theta \int_{\theta'} \nu_{t+1}^l(\theta')\, \Pi(\theta'|\theta)\, \ell_{t+1}(\theta) + L_{t+1}^k(1 + r_{\ell,t+1}) - B_{t+1}(1 + r_{t+1}),$$

where $\nu_{t+1}^l(\theta') = m_{t+1}(\theta') + p_{t+1}(\theta')$ and $\Pi(\theta'|\theta)$ is the endogenous transition probability governed by exogenous household characteristics and choices.

Banks can default at the beginning of a period by stealing a fraction ξ of their assets and not paying back their creditors. When it does so, it is excluded from banking operations in the future but can save at rate r_t. We denote the bank's value of default by $\tilde{\Psi}_{t+1}^D\left(\xi L_{t+1}'\right)$, where $L_{t+1}' = \left(\int_\theta \int_{\theta'} \nu_{t+1}^l(\theta')\, \Pi(\theta'|\theta)\, \ell_{t+1}(\theta) + L_{t+1}^k(1 + r_{\ell,t+1})\right)$. The expression $L_{t+1} = L_{t+1}^k + \int_\theta p_t(\theta)\, \ell_{t+1}(\theta)$ is the investment in t, and L_{t+1}' is the value of that investment in period $t+1$ after returns are realized. Investors lend to the bank up to a point where the bank does not default in equilibrium. Denoting the value to the bank of honoring its obligations by $\Psi_{t+1}(L_{t+1}, B_{t+1})$, the enforcement constraint is then given as $\Psi_{t+1}(L_{t+1}, B_{t+1}) \geqslant \tilde{\Psi}_{t+1}^D\left(\xi L_{t+1}'\right)$.

The bank does not face any uncertainty in its net worth even though each mortgage is risky because we assume a continuum within each household type, which translates into a continuum within each mortgage type θ.[14] Thus, an important property of the bank's problem is that all assets have to generate the same rate of return $r_{\ell,t+1}$. Therefore, $p_t(\theta) = \frac{1}{1+r_{\ell,t+1}} \int_{\theta'} \nu^l_{t+1}(\theta') \Pi(\theta'|\theta)$ for all θ; that is, the price of each mortgage is the expected present discounted value of its mortgage payments.

Since the bank is indifferent between investing in any asset, we do not have to keep track of its asset distribution in the bank's problem. Then, using $p_t(\theta) = \frac{1}{1+r_{\ell,t+1}} \int_{\theta'} \nu^l_{t+1}(\theta') \Pi(\theta'|\theta)$, we can show that $L'_{t+1} = (1 + r_{\ell,t+1}) L_{t+1}$ and that the bank's enforcement constraint becomes $(1 - \phi_{t+1})(1 + r_{\ell,t+1}) L_{t+1} \geqslant (1 + r_{t+1}) B_{t+1}$, which puts an endogenous upper bound on bank leverage.[15] This leverage constraint states that the bank can borrow up to a fraction of its assets and ϕ_{t+1} reflects the *haircut* on its collateral.[16]

The solution to the bank's problem is given as $L_{t+1} = \beta_L \widehat{\lambda}_t \omega_t$ and $B_{t+1} = \beta_L(\widehat{\lambda}_t - 1)\omega_t$, where $\widehat{\lambda}_t = \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}$. Perfect competition among banks implies that the present value of mortgage payments should be equal to the loan amount $dq^m(d; a', h, z, j) = \frac{1}{1+r_{\ell,t+1}} \int_{\theta'} \nu^l_{t+1}(\theta') \Pi(\theta'|\theta)$ at the time of the mortgage initiation. Given d and m, this equation solves for $q^m(d; a', h, z, j)$.

2.5 Symmetric Equilibrium

We focus on a symmetric equilibrium in which each bank holds the market portfolio of mortgages. Since each bank's optimal consumption and investment choices are linear in its net worth, we obtain aggregation and can focus

---

[14] Even if a bank invests in a θ-type households' mortgage by a tiny amount, its return is deterministic since a known fraction of θ-type households default. The continuum assumption grants us tractability while keeping the rich heterogeneity in the household sector.

[15] In Appendix I, we provide the characterization of the bank's problem in detail.

[16] The term ϕ_t is defined recursively as follows: $\phi_t = \xi^{1-\beta_L}\left((1 + r_{t+1}) / (1 + r_{\ell,t+1}) - (1 - \phi_{t+1})\right)^{\beta_L}$. If the bank was not able to steal (i.e., $\xi = 0$), then $\phi_t = 0$ and the *collateral premium* (or, equivalently, the credit spread) $r_{\ell,t+1} - r_{t+1}$ would be zero.

on the representative bank. In equilibrium, all economic agents maximize their objectives given bank funding cost r_{t+1} (assumed to be constant at r) and endogenous price sequences $\{r_{\ell,t}, r_{k,t}, \bar{w}_t, p_t^h, p_t^r\}_{t=1}^{\infty}$. The credit market conditions (i) $L_{t+1} = \mu w(\bar{w}_t, u_t) + \int_\theta p_t(\theta)\Gamma_t(\theta)$ determines $r_{\ell,t+1}$, and (ii) $A_{t+1} = K_{t+1} + V_{t+1}^{rc}(H_t^r)$ determines $r_{k,t+1}$. Appendix H presents all the equilibrium conditions.

Finally, note that a bank is a leveraged investor. Banks borrow the amount $\widehat{\lambda}_t - 1$ per unit of their net worth and earn an excess return $r_{\ell,t+1} - r$ on this amount in addition to $r_{\ell,t+1}$ they earn from their own net worth. Thus, a banker's gross return on net worth at time t is equal to $1 + r_{\ell,t+1} + (\widehat{\lambda}_t - 1)(r_{\ell,t+1} - r)$. In the steady state, we have $1 + r_\ell + (\widehat{\lambda} - 1)(r_\ell - r) = 1/\beta_L$, which we use with the excess return $r_\ell - r$ and the leverage rate $\widehat{\lambda} - 1$ from the data to pin down the banker's discount rate in the calibration section.

3 Calibration

A model period is two years. Households start the economy at age 25, work until they retire at age 65, and live until age 85. Table I presents externally set and internally calibrated parameters under the columns labeled "External" and "Internal" respectively.

Preferences: We assume that households receive utility from consumption and housing services captured by the following CES utility specification: $u(c,s) = \left((1-\gamma)c^{1-\epsilon} + \gamma s^{1-\epsilon}\right)^{\frac{1-\sigma}{1-\epsilon}} / (1-\sigma)$. We choose $\epsilon = 1$, which implies a unit elasticity of substitution between housing and consumption, consistent with the estimates in Piazzesi et al. (2007). Following the literature, we set $\sigma = 2$, which implies an elasticity of intertemporal substitution of 0.5.[17] We calibrate γ to match the share of housing services in GDP as 15 percent and the discount factor β to match the capital-output ratio of 1 in our biennial model.

Income Process: For the income process before retirement, we set the persistence parameter $\rho = 0.92$ and $\sigma_\varepsilon = 0.31$, which correspond to an annual

[17]Through a series of robustness exercises, we have found that the choice of ϵ and σ does not affect results significantly (Section 5.3).

Table I – Parameters (externally set and internally calibrated)

Parameter	Explanation	Value External	Value Internal
σ	risk aversion	2	
α	capital share	0.3	
ψ	curvature on hours	0.5	
ρ_ε	persistence of income	0.92	
σ_ε	std of innovation to AR(1)	0.31	
φ_h	selling cost for a household	7%	
φ_e	selling cost for foreclosures	25%	
ζ	fixed cost of mortgage origination	2	
δ_h	housing depreciation rate	5%	
τ	variable cost of mortgage origination	0.75	
η	rental adjustment cost	3	
π	prob. of being an active renter	0.265	
ι	down payment requirement	0	
β	discount factor		0.88
$\underline{h}$	minimum house size		0.69
r	deposit rate		6.47%
γ	weight of housing services in utility		0.18
$\bar{H}$	housing supply		0.93
μ	share of wage bill financed from banks		0.81
β_L	bank discount factor		0.73
ξ	bank seizure rate		0.25
κ	rental maintenance cost		0.03
δ_k	capital depreciation rate		0.20

Table II – Moments

Statistic	Data	Model
Capital-output ratio	1	1
Homeownership rate–aggregate	66%	66%
Homeownership rate-below 40	42%	42%
Debt-GDP ratio	40%	40%
House price-GDP ratio	0.825	0.825
Share of housing services in GDP	15%	15%
Ratio of mortgage loans to total loans in bank assets	0.45	0.45
Mortgage premium	0.03	0.03
Bank leverage ratio	10	10
House price-rental price ratio	5.5	5.5
Non-residential investment-output ratio	20%	20%

persistence of 0.96 and a standard deviation of 0.17 following Storesletten et al. (2004). Retirement income approximates the US retirement system, as in Guvenen and Smith (2014).

Production Sector: We set the capital share in output to $\alpha = 0.3$. Denoting Y as the final good or output, we target a capital-output ratio of $\frac{K}{Y} = 1$, which corresponds to a capital-output ratio of 2 annually.[18] We normalize $N = 1$, and $\mathbb{Z} = 1$ and target $u = 1$ at the steady state. Then, since $Y = \mathbb{Z}K^{\alpha}(Nu)^{1-\alpha}$, we get $Y = K = 1$.

The share of housing services in GDP is 0.15. Since in our model, GDP, which includes the imputed income from housing, corresponds to $Y_A = Y + p_r\bar{H}$, this results in $Y_A = \frac{1}{0.85}$ and $p_r\bar{H} = \frac{0.15}{0.85}$. In the data, the ratio of non-residential investment to GDP is 0.2. Since, at the steady state, this ratio is $\frac{\delta_k K}{Y}$, this gives us a capital depreciation rate of $\delta_k = 0.2$ biennially. Given these targets, the model-implied biennial return to capital becomes $r_k = \alpha\frac{Y}{K} - \delta_k = 10$ percent.

We calibrate the labor utilization function curvature ψ to match the response of hours in the model to the data. We choose $\psi = 0.5$ with which the model generates an employment decline of 1.8 percent in response to a 1 percentage point increase in the bank lending rate, which falls in the middle of the employment effect found in Gertler and Gilchrist (2018). We target $u = 1$ in the steady state. From the firm's problem, $\vartheta = \left(\frac{1-\alpha}{1+\mu r_\ell}\right)\left(\frac{\alpha}{r_k+\delta}\right)^{\frac{\alpha}{(1-\alpha)}}$ gives the calibrated value of ϑ.

Housing Market: The probability of an inactive renter becoming an active renter is set to 0.265 to capture the fact that the bad credit flag remains for about seven years in the credit history of households. We set the selling cost (φ_s) to 7 percent for regular sales and to 25 percent for foreclosed properties, consistent with the estimates of Campbell et al. (2011). We set the fixed mortgage origination cost $\zeta = 2$ percent of GDP and the variable cost of mortgage origination $\tau = 0.75$ percent of the mortgage loan (Federal Reserve Board (2008)). We set the down payment requirement to zero since there is no

[18]This implies a capital-to-GDP ratio of 1.7.

explicit regulation for down payment. However, in the model many households choose to make some down payment in order to get favorable mortgage terms.

The ratio of house prices to biennial rental payments is 5.5 (Sommer et al. (2013)). This moment, together with the fact that the ratio of housing services to GDP is 0.15, implies $\frac{p_h \bar{H}}{Y_A} = \frac{p_r \bar{H}}{Y_A} \times \frac{p_h}{p_r} = 0.15 \times 5.5 = 0.82$. So, we set $\bar{H}$ to match this ratio. We set the biennial depreciation rate for housing units as $\delta_h = 5$ percent (Harding et al. (2007)). The steady-state relation between the rental price and house price is given by $p_r = \kappa + \frac{r_k + \delta_h}{1 + r_k} p_h$. This gives us an estimate of κ given our target $\frac{p_h}{p_r} = 5.5$. We restrict the minimum house size for owner-occupied units to be $\underline{h}$ to match a homeownership rate of 66 percent. Lastly, we choose $\eta = 3$; however, our results turn out to be not sensitive to the particular value of η (see Section 5.3 and Appendix D).

Financial Sector: Since not only banks but also other institutions hold large amounts of mortgage-related products, we follow Shin (2009) and include deposit-taking institutions (US chartered depository institutions and credit unions), issuers of asset backed securities, GSEs, and GSE-backed pools from FED Z1 data in our bank definition. Then we match bank balance sheets to the 1985-1994 average in the data. We use Tables L.218 and L.219 to obtain the total amount of home and multifamily residential mortgages held by banks. Banks on average hold \$2.117 trillion of these mortgages, which correspond to 86 percent of all mortgages (stable from 1985 to 1994). To compute the amount of lending to non-financial firms, we use the balance sheets of non-financial firms (Table L.102). We use total loans (loans from depository institutions, mortgages, and other loans), which average to \$2.245 trillion, and miscellaneous liabilities, which average to \$1.23 trillion. Residential mortgages constitute 49 percent of banks' balance sheets if we include the loans only and 39 percent if we also include miscellaneous liabilities as firms financing from banks. Thus, we choose $\frac{\int_\theta p_t(\theta)\Gamma_t(\theta)}{\mu w(\bar{w}_t, u_t) N_t + \int_\theta p_t(\theta)\Gamma_t(\theta)}$ (the ratio of mortgages to banks' total financial assets) as 45 percent, which gives μ, the fraction of wage bill financed through banks.

In the steady state, we have $r_\ell - r = \frac{1 - \beta_L(1+r)}{\widehat{\lambda}\beta_L}$, where $\widehat{\lambda} = \frac{(1+r)}{1 + r - (1-\phi)(1+r_\ell)}$

is the endogenous leverage ratio and $\phi = \xi^{1-\beta_L} \left({(1+r)}/{(1+r_\ell)} - (1-\phi) \right)^{\beta_L}$ is the haircut. We calibrate r to match a debt-GDP ratio of 40 percent (corresponding to an 80 percent ratio annually), and we target $r_\ell - r = 3$ percent, representing the average biennial gap between the 30-year mortgage interest rate and the 10-year Treasury rate in the data. We also target the bank leverage ratio $\widehat{\lambda}$ as 10 following Gertler and Kiyotaki (2015). These two targets give us the bank's discount factor β_L and the bank's seizure rate ξ.

Overall, we have 10 parameters that we calibrate internally: discount factor (β), minimum house size ($\underline{h}$), deposit rate (r), weight of housing services in utility (γ), housing supply $\bar{H}$, share of wage bill financed by banks (μ), bank's discount factor (β_L), bank's asset seizure rate (ξ), maintenance cost for rental units (κ), and capital depreciation rate (δ_k). The last four of these parameters are identified directly through analytical moments obtained through the model as discussed above. This leaves us with six parameters that we calibrate using the model simulated data to jointly match the following six data moments (Table II): 66 percent average homeownership rate, 42 percent homeownership rate under the age of 40, 40 percent mortgage-debt-to-GDP ratio, capital-output ratio of 1, share of mortgages in bank balance sheet as 45 percent, and share of housing services in GDP as 15 percent.

Leverage Shock: The boom and bust periods coincided with important changes in financial markets that shifted credit supply.[19] The Glass-Steagall Act, the bill that separated banking activities from investment banking ones, after being loosened for about a decade, was repealed in 1999. As a result, deposit-taking banks had the opportunity to extend their balance sheets. On the securitization side, from 1995 to 2005, the volume of private-label mortgage-backed securities increased dramatically from negligible levels to $1.2 trillion but disappeared with the crisis. We view both the regulatory changes and changes in investor sentiment toward mortgage-backed securities as the driving force behind the expansion and then contraction of funding to the banking

[19] See Chernenko et al. (2014) for developments in the securitization market and Sherman (2009) for important changes in financial market regulation in the US.

system. Nevertheless, the distinction between the two is not critical for our analysis since the model is calibrated to match the increase in bank leverage until 2008 without taking a stand on the underlying reason.

To study the role of bank credit supply, we assume that the economy is at the steady state before 1998, but in 1998, unexpectedly, bank leverage starts gradually increasing for 20 years. Unexpectedly, in 2008, however, the leverage reverts back. In the model, the bank leverage is determined by ξ: a lower (higher) value for ξ reflects higher level of (lower) trust for banks by creditors and allows banks to have a higher (lower) leverage. To calibrate the changes in this parameter, we refer to two sources. First, Federal Reserve Bank of New York (2020) documents that the leverage ratio of the consolidated US banking organizations has increased by 25 percent from the first quarter of 1996 to the last quarter of 2007. We use the leverage ratio of all institutions (see page 34 of the report). Second, the Financial Stability Report by Federal Reserve Board (2019) documents that the leverage ratio of security brokers and dealers has increased by 50 percent from the first quarter of 1995 to the first quarter of 2008 (see Figure 3-5 in the report).

Both studies report marked-to-book leverage. However, in our model bank assets, L_{t+1}, and net worth, ω_t, are in market values and the ratio L_{t+1}/ω_t gives the marked-to-market leverage, which is the same as the book leverage when the economy is in steady state. However, after unexpected shocks, market and book values will no longer be equal. To be able to compare the model to the data, we compute the book values of bank loans and net worth and calculate the corresponding book leverage in our model. We calibrate the changes in parameter ξ to have an increase in the financial system book leverage for 37.5 percent (from 1996 to 2006), which falls in the mid-range of 25 percent and 50 percent. We impose a linear change in ξ over 20 years. The top left panel in Figure 3 compares the book leverage from our model and these sources.[20]

[20] In our framework, the leverage constraint and haircuts on collateralized loans are tightly linked. Available data suggest that haircuts more than doubled for most mortgage-related securities after the crisis (Committee on the Global Financial System (2010)), consistent with the leverage dynamics in our model.

Housing Demand Shock: The boom leverage shock generates part of the housing boom. We generate the remainder of the housing boom with a housing demand shock in the spirit of Kaplan et al. (2020). We assume that homeowners receive a premium χ_t in $u((1+\chi_t)c, (1+\chi_t)s)$ with $\chi_t = 0$ in the initial steady state. We calibrate the increase in χ_t to match the remainder of the house price boom in the data so that both shocks together generate a 28 percent increase in house prices during the boom. Then, χ_t unexpectedly reverts to its initial steady-state value of zero in 2008. Similar to the leverage shock, starting in 1998, we impose a linear change in the homeownership premium parameter over 20 years. The calibration results in an increase of 0.1 in χ_t over time.

Government interventions at the bust: Even though both shocks return back to their initial steady state levels in 2008, the model economy experiences a large bust, in which bank net worth becomes negative in the absence of any government intervention. This result suggests that the government interventions were necessary to keep the banking sector afloat in the crises. In order to incorporate interventions into our framework that are consistent with the actual experience, we assume that the government borrows from international investors at the rate r in 2008 to finance bailouts to households and banks and rolls over its debt until 2038, after which it increases the labor income tax to balance the government budget in the long run.[21] For the household bailout, we assume that the government pays banks for the portion of a household's debt that is above a threshold leverage ratio. We choose the threshold to match the 3.6 percent increase in the foreclosure rate during the bust. The calibrated threshold is 117 percent, which turns out to be almost the same as the one used by the Home Affordable Modification Program (HAMP).[22] For the bank bailout, we assume that the government injects equity into banks so

[21]The particular choice of the year after which the labor income tax is increased does not affect the boom-bust dynamics we report in the paper as long as it is in the distant future. We have also experimented with no tax increase as if the bailout amount is a windfall to check the sensitivity of our results. Again, we did not discern any noticeable effect on the boom-bust.

[22]HAMP provided debt forgiveness and payment modification to households with leverage ratios above 115 percent. See Ganong and Noel (2020) for the details of these programs.

that the bank net worth ω_t declines by 67 percent in 2008, consistent with the decline in market equity of bank holding companies reported in Begenau et al. (2019, Figure 3, left panel). With this intervention, the bank collateral premium or equivalently the credit spread $(r_\ell - r)$, the measure of the distress in the banking system, increases by 4.4 percentage points (top row, middle panel, Figure 3). For comparison, the excess bond premium from Gilchrist and Zakrajšek (2012) and Gertler and Gilchrist (2018) increases by about 3 percentage points during the 2008 financial crises. Finally, the total bailout amount in the model is 6.5 percent of GDP.[23]

4 Performance of the benchmark model

In this section, we compare our model's distributional and aggregate implications during the boom-bust cycle with the data.

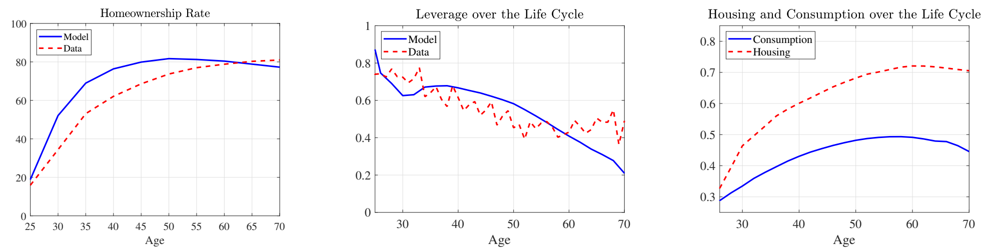
4.1 Distributional Implications

The life-cycle implications of the model closely match the data (Figure 1). The homeownership rate increases over the life cycle. Leverage declines with age, but more than the data after age 60. Average consumption and housing increase over the life cycle, consistent with the findings of Aguiar and Hurst (2013).

The left panel of Figure 2 shows that the credit shares of each income quantile remained stable from 1996 to 2006, consistent with the evidence in Adelino et al. (2016), Albanesi et al. (2017), and Foote et al. (2016). The middle panel shows that household leverage is higher in 2006 than 1996 in

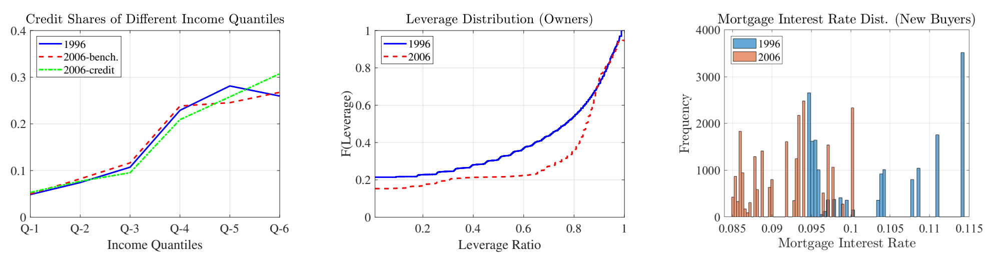
[23]The US Congress approved \$700 billion as part of the Troubled Asset Relief Program (TARP) in 2009. However, only \$475 billion of this amount was actually used. Private sector GDP in 2009 was \$11.8 trillion (\$14.7 trillion total value added minus \$2.9 trillion value added by the government). The amount of \$700 (\$475) billion is 6 (4) percent of GDP. Even though the bailout amount is larger in the model than in the data, remember that we have only two fiscal interventions and abstract from the monetary policy response since the effectiveness of monetary interventions depends crucially on people's expectations about the future policy path, which makes our decomposition exercises blurry and significantly more complicated.

Figure 1 – Life-Cycle Properties: Model versus Data



Notes: The graph shows the life-cycle properties of housing and mortgage debt. The left panel plots the homeownership rate. The middle panel plots mortgage debt relative to housing value. The data come from the 1995 Survey of Consumer Finances. The right panel plots consumption and housing expenditure over the life cycle from the model.

Figure 2 – Cross-sectional Developments in Credit and Housing during the Boom



Notes: The graph plots several model-implied cross-sectional implications. The left panel plots the mortgage credit distribution across income quantiles in 1996 and 2006 for both the benchmark and the result from the model with only the credit supply shock. The middle and right panels plot the distributions of the leverage ratio and interest on mortgages in 1996 and at the peak of the boom in 2006.

the model, in part because the model generates endogenous increases in LTV ratios consistent with the data without appealing to exogenous changes in LTV limits.[24] Finally, the right panel shows that interest rates on mortgages declined from 1996 to 2006, as in the data (Favilukis et al. (2017)).

4.2 Boom-Bust Cycle in Aggregates

Before we present the performance of the model in terms of matching the boom-bust dynamics of important aggregates around 2008, it will be

[24] Keys et al. (2012) report that the average combined LTV of households increased by 15 percentage points during the crises, whereas our model implies a 9 percentage points increase.

instructive to illustrate how leverage and housing demand shocks transmit to the economy. Figure B.1 in Appendix B.1 illustrates linkages across sectors and the transmission mechanisms of shocks.

Transmission of the leverage shock: The changes in the bank lending rate r_ℓ is the key mechanism through which the leverage shock transmits to the economy. A lower r_ℓ, for example, implies lower borrowing costs for both households and firms. Then, households' demand for consumption and housing increases, which is the *direct* effect of lower r_ℓ. Furthermore, firms demand more labor and the equilibrium wage increases, and hence households demand more consumption and housing, which is the indirect effect of lower r_ℓ.

Exogenous shocks to bank leverage translate into one-for-one changes in the bank lending rate r_ℓ in the absence of any feedback from bank balance sheets. Focusing first on the steady state, an increase in bank leverage decreases the credit spread according to $r_\ell - r = \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$. Thus, a permanent increase in $\widehat{\lambda}$ will lead the economy to a steady state with a lower r_ℓ. Moreover, when the bank net worth effects are absent, changes in $\widehat{\lambda}$ will translate into changes in r_ℓ during the transition, as given by this equation. As a result, r_ℓ gradually falls during the boom and is expected to stay low permanently.

The leverage parameter reverts back unexpectedly and permanently to its steady state level at the time of the bust, which translates into a permanent increase in r_ℓ. However, the deterioration of bank balance sheets amplifies the increase in r_ℓ during the bust. We explain this amplification mechanism next.

Although all variables affect each other simultaneously, we will proceed with an iterative approach in demonstrating the amplification mechanism during the bust. For this purpose, remember that the bank net worth in period t is given as

$$\omega_t = \int_\theta \int_{\theta'} \left(m_t\left(\theta'\right) + p_t\left(\theta'\right)\right) \Pi\left(\theta'|\theta\right) \Gamma_{t-1}\left(\theta\right) + L_t^k\left(1 + r_{\ell,t}\right) - B_t\left(1 + r_t\right).$$

The shock that generates the bust is a decrease in $\widehat{\lambda}_t$ back to its steady-state level that reduces loan supply through $L_{t+1} = \beta_L \hat{\lambda}_t \omega_t$.[25] As a result, the equilibrium bank lending rate $r_{\ell,t+1}$ increases. However, a higher $r_{\ell,t+1}$ reduces the bank's net worth at time t by lowering mortgage valuations since $p_t(\theta) = \frac{1}{1+r_{\ell,t+1}} \int_{\theta'} (m_{t+1}(\theta') + p_{t+1}(\theta')) \Pi(\theta'|\theta)$ for all θ. In response, loan supply L_{t+1} declines further and $r_{\ell,t+1}$ increases more. With higher $r_{\ell,t+1}$, mortgage valuations and bank net worth decline further, which generates further increases in $r_{\ell,t+1}$ and future bank lending rates. This is the key mechanism through which the deterioration of bank balance sheets amplifies the transmission of a shock to bank leverage. However, the spike in r_ℓ and the sharp drop in bank net worth are short-lived because the amplification mechanism described above works the opposite way in the recovery. When r_ℓ starts to decline, the market value of the bank's mortgage portfolio starts recovering. This increases the bank's net worth, hence credit supply, reducing r_ℓ's even more. As a result, bank net worth recovers quickly.

Transmission of the housing demand shock. The housing demand shock directly affects house prices, and its effect on the rest of the economy is mostly through house prices. During the boom, for example, an increase in house prices raises consumption because of wealth and collateral effects. During the bust, however, an important indirect effect of the housing demand shock arises as a result of its effect on bank balance sheets since the sharp decline in house prices increases foreclosures. The losses in bank balance sheets cause a decline in bank credit, which initiates a mechanism similar to the one above. This mechanism via banks is an important component in this paper.

4.2.1 Banking Sector Dynamics

The banking sector experiences a boom-bust cycle in the model, as in the data (Figure 3). The bank book leverage increases by 37.5 percent in the model (as it is calibrated). The leverage does not change significantly in the bust period, but it declines by 42.8 percent in the subsequent two

[25] The term $\widehat{\lambda}_t$ is an endogenous object determined by equation I.1 in Appendix I. The parameter that goes back to its steady-state level is ξ, which eventually decreases $\widehat{\lambda}_t$ to its initial steady-state level.

periods and recovers slowly. The leverages of commercial banks and security brokers and dealers in the data decline by 27 and 71 percent from 2008 to 2010, respectively.[26]

With higher leverage, the loan supply increases, and the bank lending rate r_ℓ declines. The bank net worth declines during the boom (top row, second panel) since the bank funding cost r has not changed and r_ℓ is lower. Thus, the banking sector supports more credit with lower bank net worth but with higher debt. The model generates a 29.2 percent rise in bank loans in the boom, slightly below the data. The share of mortgages in banks' portfolios also increases, as does the value of the mortgage pool.

The crisis occurs as the leverage constraint and housing demand parameters revert to their initial steady-state levels. The credit spread $r_\ell - r$ jumps by 4.4 percentage points—in line with the spike in the excess bond premium documented in Gilchrist and Zakrajšek (2012) and Gertler and Gilchrist (2018)—and mortgage valuations and bank net worth sink.[27] Since mortgages are long-term assets, banks cannot flexibly adjust their balance sheets by issuing fewer mortgages. Therefore, they reduce their lending to firms by 30 percent.

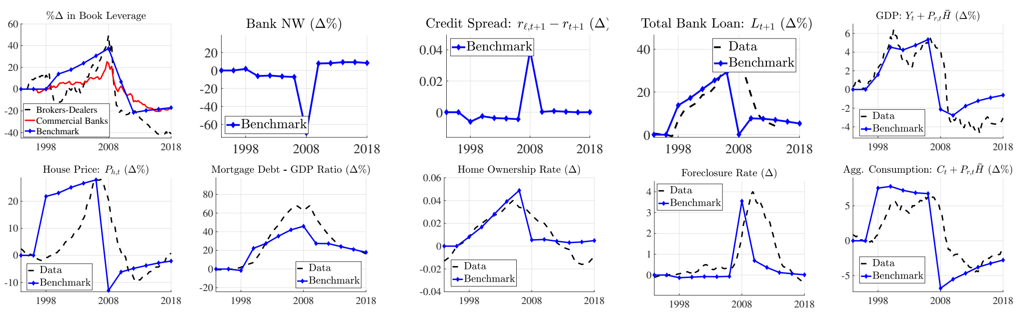
4.2.2 Real Sector Dynamics

At the peak of the boom, the US per capita GDP was almost 6 percent above its trend. GDP ($Y_A = Y + P_r \overline{H}$) in the model increases by 5.4 percent (top row, right panel in Figure 3) during the boom. With the crisis, GDP falls by 7.1 percent from its peak, almost matching the data counterpart. The aggregate labor income follows a boom-bust pattern similar to the output, which is critical

[26]Consistent with what we report here, there is broad agreement that marked-to-book leverage is procyclical (Adrian and Shin (2010), Nuno and Thomas (2017), and Coimbra and Rey (2017)). The marked-to-market bank leverage (L_{t+1}/ω_t) also increases during the boom but spikes at the time of the bust as bank net worth sinks, which is also consistent with the findings in Begenau et al. (2018).

[27]The dynamics of the bank lending rate are consistent with other empirical findings as well. During the boom, interest rates on firm loans and mortgages declined (Glaeser et al. (2012) and Justiniano et al. (2017)). Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, the lending interest rate on loans more than quadrupled (Adrian et al. (2013)).

FIGURE 3 – Boom-Bust Dynamics



Notes: The graph plots the dynamics of key variables during the boom-bust episode. Credit spread is measured annually. Data counterparts of bank loans, output, consumption, and house price are percentage deviations from their linear trends obtained from the 1985-2006 period. Total data for bank loans include home and multifamily residential mortgages, and firm loans and miscellaneous liabilities. The data for the book leverage of banks are from Federal Reserve Bank of New York (2020); data for security brokers and dealers are from Federal Reserve Board (2019). In the text, we compute the bust changes relative to the peak of the boom.

for the boom-bust dynamics in the housing market and consumption, as we analyze in Section 5.2.1.

The response of aggregate consumption ($C_A = C + P_r\overline{H}$) to the shocks is more abrupt and exaggerated compared with the data (bottom row, right panel, in Figure 3).[28] In the model, this is driven by the immediate response of house prices to new information from two unexpected shocks. Since house prices are one of the key determinants of consumption (together with labor income), their jump and fall with the boom and bust create a non-smooth consumption pattern. This undesirable feature is not unique to our model and is common in asset-pricing models that feature unexpected shocks.

The response of the firms' labor demand to the changes in the bank lending rate is the key driver of the boom-bust in the production sector (Section 5.2.1). Our model generates changes in total per capita hours worked, labor income, and firm loans that are similar to the data. Moreover, there is substantial

[28]Since the bank borrowing rate r is exogenous, this model is essentially an open economy where output is given as $Y_A = C + I + NX$. In 1998, NX declines and stays negative. Thus, the model implies that there is a net capital inflow to the US during the boom period, which is broadly consistent with the data.

evidence that the labor decisions of firms are indeed influenced by financial conditions, which corroborates the predictions of our model.[29]

4.2.3 Housing Market Dynamics

House prices increase 27.9 percent during the boom (Figure 3), as the size of the housing demand shock is chosen to match it. The surprising result is that the bust is much deeper: house prices decline by 31.9 percent from their peak to 13 percent below their initial steady-state level and slowly recover, as in the data. As a result of higher house prices, increased homeownership rate, and the jump in refinancing activity, the mortgage-debt-to-GDP ratio increases 46 percent in the model compared to 63 percent in the data during the boom. After the bust, household debt gradually declines. The foreclosure rate in the model stays low during the boom and jumps by 3.6 in the bust period (as in the data), as a more than 30 percent decline in house prices during the bust pushes many households to negative equity, which makes default an attractive option. The homeownership rate rises by 4.9 percent during the boom, which is close to the data, and declines during the bust because of the decline in housing demand and defaulting households.

5 Quantitative Analyses of the Boom-Bust Cycle

Equipped with a model that can generate economic dynamics that are close to the data, we now study the drivers of the boom-bust cycle. We want to highlight two points about the decomposition exercises. First, if we add different mechanisms sequentially on top of each other to measure their relative contributions, the order of decomposition matters. Since there are many possible orderings, we evaluate the contribution of each mechanism by closing all others and adding that mechanism only.

[29] For example, Chodorow-Reich (2013) finds that firms that worked with weaker banks prior to the crisis reduced employment more. Benmelech et al. (2019) find similar evidence from the Depression era, and Popov and Rocholl (2015) bring evidence from Germany during the 2008 crisis. Finally, Ivashina and Scharfstein (2010) document a more than 50 percent decline in bank capital expenditure and working capital loans to corporations, and Adrian et al. (2013) find that capital expenditure and working capital loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis.

Second, we need to make sure that our decomposition exercises are not affected by the existence of bailouts. To do that, we keep all parameters and other factors at their boom levels, including the bailout amount which is zero, and compute the size of the bust generated by only one factor without any bailouts. Since there are no bailouts in the boom and in this counterfactual experiment, the change in a variable from the boom to the bust gives the contribution of that factor only.

5.1 The importance of leverage and housing demand shocks

Changes in credit supply and housing demand have been proposed as two competing explanations for the boom-bust cycle in the US housing market. In this section, we quantify the importance of the bank leverage shock (one source of credit supply change) and the housing demand shock to the boom-bust. For this, we solve the equilibrium transition of the model with only one "boom" shock (bank leverage or housing demand) and compute the size of the boom generated by that shock only. We report the results of these exercises under "Boom/Only/Δx_{LS}" and "Boom/Only/Δx_{HDS}" in Table III. We also report the changes in variables from our benchmark under "Boom/Benchmark/$\Delta x_{(LS+\ HDS)}$" for comparison.

Focusing on the boom, we find that the leverage shock by itself generates a 9.8 percent increase in house prices and virtually all of the decline in the bank lending rate and most of the increase in wages and output. Consumption, driven by house prices and wages, increases by 4.9 percent. The housing demand shock generates a larger increase in house prices (15.4 percent). However, it does not significantly affect the bank lending rate and output and has only a small effect on consumption (0.8 percent).

To assess the contributions of each of these shocks to the bust, we hit the economy with one bust shock (bank leverage or housing demand) while the economy is at the peak of the benchmark boom generated by both shocks (the "Bust" column in Table III). Each shock alone generates a large bust in the

Table III – Contributions of shocks to leverage (LS) and housing demand (HDS) to boom-bust

	Boom			**Bust**		
	Benchmark	Only		Benchmark	Only	
	$\Delta x_{(\text{LS + HDS})}$	Δx_{LS}	Δx_{HDS}	$\Delta x_{(\text{LS + HDS +Bailout})}$	Δx_{LS}	Δx_{HDS}
Bank lending rate (Δr_ℓ)	-0.5	-0.5	0.0	4.4	4.6	3.2
Output (%ΔY)	4.7	4.3	-0.0	-5.3	-5.6	-3.7
Wages (Δ%w)	5.4	5.0	-0.1	-11.5	-12.1	-8.4
Consumption (%ΔC)	6.1	4.9	0.8	-11.5	-9.9	-10.1
House price (%Δp_h)	27.9	9.8	15.4	-31.9	-17.8	-30.9
Foreclosure rate (ΔFr)	0.0	0.0	0.0	3.6	0.7	11.5

Note: The table reports the results where we decompose the role of leverage and housing demand shocks. "LS" refers to leverage shock, "HDS" refers to housing demand shock. "Only" columns report the results with only one of the shocks. The percent decline in the bust is calculated with respect to the value at the peak of the boom.

economy. In fact, the sum of the busts generated by each shock is larger than the benchmark bust. However, note that the government bailouts significantly mitigate the bust. Thus, our analysis suggests that the crisis would be much more severe in the absence of these bailouts.[30]

We find large effects for both leverage and housing demand shocks during the bust. The bank lending rate increases by 4.6 and 3.2 percentage points, and output declines by 5.6 and 3.7 percent on impact, with leverage and housing demand shocks, respectively. Even if the decline in output is smaller on impact under the housing demand shock, the worsening of household balance sheets generates a subsequent 4.6 percent decline as a result of lower capital accumulation by households. The housing demand shock generates a bigger

[30]We do not report the amount of bust mitigated by the bailouts. In principle, we could compute the amount of bust generated by both shocks without bailouts, call it $\Delta x_{\text{LS+HDS}}$. Then the difference between the benchmark and the bust generated by this exercise would be the effect of bailouts: $\Delta x_{\text{bailout}} = \Delta x_{(\text{LS + HDS +Bailout})} - \Delta x_{(\text{LS+HDS})}$. However, as we noted earlier, the bank net worth becomes negative in the bust in the absence of bailouts. Figuring out how the economy evolves from that point generates further complications to our analyses. Thus, we cannot obtain $\Delta x_{\text{LS+HDS}}$. An alternative is to compute the effect of the bailouts as $\Delta x_{\text{bailout}} = \Delta x_{(\text{LS + HDS +Bailout})} - (\Delta x_{\text{LS}} + \Delta x_{\text{HDS}})$, which can be read as the residual from Table III. To the extent that $\Delta x_{\text{LS}} + \Delta x_{\text{HDS}} = \Delta x_{\text{LS+HDS}}$, these two exercises would give similar results. However, this equality does not necessarily hold since the ordering of decomposition matters.

decline in house prices (30.9 versus 17.8) and a larger increase in the foreclosure rate (11.5 versus 0.7 percentage points) relative to the leverage shock. The disproportionately larger impact of the housing demand shock on foreclosures is mainly because it reduces the ownership benefit and makes foreclosure a more attractive option for a given house price decline. Finally, the decline in consumption is driven by the declines in house prices and income, and the leverage shock generates a 9.9 percent decline while the housing demand shock generates a 10.1 percent decline.

The housing demand shock affects all aggregates more during the bust than the boom because the decline in house prices increases foreclosures and hurts bank balance sheets. The resulting decline in credit supply causes the equilibrium bank lending rate to increase by 3.2 percentage points and wages to decline by 8.4 percent. Thus, the role of the housing demand shock is amplified by the bank balance sheet deterioration, which we further investigate in Section 5.2.3.

We would like to conclude this section with two remarks. First, while we find that the housing demand shocks generate a large house price boom-bust, as in Kaplan et al. (2020), different from them we also find large effects of the credit supply on house prices. Our analysis differs from theirs in two aspects. First, the credit supply shock in our framework is not an isolated shock to the household due to the interaction between the bank balance sheets and firms' production. As we show in Section 5.2.1, the changes in credit supply—due to exogenous shocks to bank leverage and/or endogenous changes in bank balance sheets—generate changes in the bank lending rate, which affect households both directly through their borrowing cost and indirectly through firm borrowing costs. Second, we consider permanent changes in bank leverage that translate into permanent changes in the bank lending rate rather than the LTV, PTI, or temporary interest rate shocks considered in Kaplan et al. (2020). Changes in LTV and PTI requirements shift the housing demand between renting and owning. Hence, these shocks do not significantly affect the aggregate housing demand. Moreover, because of endogenous borrowing limits on households, LTV and PTI limits become almost irrelevant.[31] On the other hand, a permanently lower bank lending rate increases the total housing demand because it creates a positive income effect—since mortgage payments decline for a given debt amount—and a positive wealth effect—since labor income permanently increases.

Second, Adelino et al. (2016), Albanesi et al. (2017), and Foote et al. (2016) find that credit grew uniformly across income groups during the boom period, a result that has been interpreted as evidence against the credit supply channel since the previous literature considered the credit supply channel to be more relevant for lower-income households. We show that in fact the bank leverage shock alone can generate uniform credit growth across income groups (left panel of Figure 2). Thus, our analysis shows the importance of structural modeling in interpreting the data.

5.2 Feedback from bank balance sheets to household and firm borrowing rates

The changes in credit supply—due to exogenous shocks to bank leverage and/or endogenous changes in bank balance sheets—generate changes in the bank lending rate, which affect households both *directly* through their borrowing cost and *indirectly* through firm borrowing costs. In this section, we quantify the importance of the changes in credit supply through a series of experiments and decompose their effects into direct and indirect effects.

In the first experiment, we quantify the amount of boom-bust accounted for by the changes in the bank lending rate $\mathfrak{r}_\ell$ in the benchmark economy. The changes in $\mathfrak{r}_\ell$ in this exercise have permanent components due to exogenous and permanent changes in bank leverage in the boom-bust and also a temporary

[31] Relaxing these constraints does not generate any boom in our model. To check whether tightening them generates a bust, we have conducted an experiment in which the LTV limit is reduced from 100 percent to 80 percent, and a PTI ratio of 25 percent is imposed (the benchmark does not have a PTI limit) with 0.6 persistence. Despite their large sizes, the LTV shock generates a small effect on aggregates, while the PTI shock has almost no effect (see Figure C.2 in Appendix C).

spike at the time of the bust due to the deterioration of the bank balance sheet. In the second experiment, we quantify the contribution of the endogenous bank balance sheet deterioration to the bust, which manifests its effect as the temporary spike in the bank lending rate. In the third experiment, we explore how much the effect of the housing demand shock during the bust is amplified through the bank balance sheet deterioration, and hence, the endogenous decline in credit supply.[32]

5.2.1 The Roles of Bank Lending Rate and Labor Income in the Boom-Bust

To analyze the role of the changes in bank lending rate r_ℓ in the boom-bust cycle, we compute the amount of boom-bust generated if all shocks are shut down and the benchmark r_ℓ sequence in Figure 3 is fed into the economy (as two unexpected shocks) and all other prices are solved endogenously. Results are under the "total r_ℓ effect" column in Table IV.

During the boom, the decline in r_ℓ (expected to be permanent) reflects the exogenous increase in bank leverage since the endogenous changes in bank balance sheets have a minimal effect. The total contribution of r_ℓ to the boom is very similar to the contribution of the leverage shock because the leverage shock accounts for all of the decline in r_ℓ during the boom. Overall, the decline in r_ℓ generates increases of 4.7, 9.3, and 4.7 percent in wages, house prices, and consumption, respectively (the leverage shock generates increases of 5.0, 9.8 and 4.9 percent, respectively).[33]

Then we generate an r_ℓ-induced bust by increasing r_ℓ unexpectedly (to the benchmark bust sequence) while the economy is at the peak of the benchmark boom. The total effect of r_ℓ causes a 11.5 percent decline in wages (same as the

[32]We do not present the bank balance sheet amplification arising from the leverage shock as a separate experiment since the spikes in the bank lending rate from the benchmark and from the leverage shocks are quantitatively similar. Thus, the bank balance sheet effects from the leverage shock will be similar to the second experiment.

[33]The small difference is because of the housing demand shock, which affects the benchmark r_ℓ dynamics relative to an economy with a pure leverage shock, as analyzed in the previous section.

TABLE IV – Roles of Bank Lending Rate and Labor Income

	Benchmark	Total	Total r_ℓ effect decomposed	
	Boom	r_ℓ effect	r_ℓ direct	wage effect
Bank lending rate (Δr_ℓ)	-0.5	-0.5	-0.5	–
Wages ($\%\Delta w$)	5.4	4.7	–	4.7
House price ($\%\Delta p_h$)	27.9	9.3	4.9	3.9
Consumption ($\%\Delta C$)	6.1	4.7	0.6	4.3
	Bust			
Bank lending rate (Δr_ℓ)	4.4	4.4	4.4	–
Wages ($\%\Delta w$)	-11.5	-11.5	–	-11.5
House price ($\%\Delta p_h$)	-31.9	-16.2	-7.8	-6.3
Consumption ($\%\Delta C$)	-11.5	-9.3	-1.5	-7.6

Notes: This table reports results on the role of bank lending rate r_ℓ on house prices and consumption for the boom and bust separately. The "Total r_ℓ effect" column reports the results where the benchmark r_ℓ sequence in Figure 3 is fed into the economy (all prices are allowed to react but parameters are fixed). The "r_ℓ direct" column reports the results where all prices (except the house prices) and parameters are fixed. The "wage effect" column reports the results where the wage sequence from the "Total r_ℓ effect" exercise is fed into the economy while keeping the interest rate, the return on capital, and the parameters unchanged. The p_h and p_r are linked by the formula driven from the first order condition of the rental company. Only focusing on changes in p_h keeping p_r constant does not significantly alter the results.

benchmark decline), which shows that the spike in r_ℓ is the primary cause of the decline in wages at the time of the bust. However, the persistence of the wage decline is smaller under the r_ℓ shock than the benchmark since the housing demand shock in the benchmark further deteriorates household balance sheets and causes larger subsequent declines in capital. The total effects of r_ℓ on house prices and consumption are declines of 16.2 and 9.3 percent, respectively.[34]

[34]Notice that the effect of r_ℓ is very similar to the effect of the leverage shock during the bust, but this does not have to be the case since the housing demand shock as well as bailouts play significant roles on r_ℓ . However, the increase in r_ℓ from only the leverage shock is 4.6 percentage points, which turns out to be very similar to the benchmark increase in r_ℓ, which is 4.4 percentage points.

The changes in the bank lending rate affect households both *directly* via borrowing costs and *indirectly* via firms' labor demand and thus labor income. Under the column "r_ℓ direct" we measure the direct effect of r_ℓ on the boom by feeding into the economy the r_ℓ boom sequence (a decline of 0.5 percentage points) keeping all parameters, the wage rate w and the return on capital r_k fixed at their steady-state levels. Under the column "wage effect" we measure the indirect effect of r_ℓ by feeding into the economy the boom wage sequence (obtained from the "Total r_ℓ effect" exercise), keeping parameters, r_ℓ and r_k at their steady-state levels. In both exercises, we solve house prices endogenously. We follow the same methodology and terminology in Tables V and VI in Sections 5.2.2 and 5.2.3 as well.

These exercises show that the permanent decline in r_ℓ can generate a 4.9 percent increase in house prices directly through household borrowing costs and an additional 3.9 percent increase by indirectly affecting firm borrowing rates and hence labor income. The indirect effect of r_ℓ on consumption turns out to be much more important than its direct effect (4.3 percent versus 0.6 percent) since the changes in consumption are driven by the changes in house prices and labor income.

We reach a similar conclusion for the bust period. While the dynamics of the bank lending rate alone can *directly* generate a 7.8 percent decline, they can generate another 6.3 percent decline in house prices *indirectly* via wages. For consumption, the changes in the bank lending rate directly generate a 1.5 percent decline, and wages indirectly reduce it by an additional 7.6 percent, accounting for most of the consumption decline.

5.2.2 The Role of Bank Balance Sheet Deterioration in the Bust

The increase in the bank lending rate after the bust reflects both the exogenous tightening of the bank leverage constraint and the temporary endogenous contraction of the bank credit supply due to the bank balance sheet deterioration in the bust period. Here, we focus on the latter and quantify the amplification generated by the bank balance sheet deterioration only, which is reflected as the 3.9 percentage points spike on top of the permanent 0.5

Table V – The Role of the Bank Balance Sheet Deterioration in the Bust

Variables	Benchmark Bust Δ%	Total effect of the spike in r_ℓ	Effect of the spike decomposed: r_ℓ direct	wage effect
Bank lending rate (Δr_ℓ)	4.4	3.9	3.9	–
Wages ($\%\Delta w$)	-11.5	-10.1	–	-10.1
House price ($\%\Delta p_h$)	-31.9	-9.9	-4.7	-3.6
Consumption ($\%\Delta C$)	-11.5	-6.3	-1.0	-5.3

Notes: This table reports results on the role of the spike in the bank lending rate r_ℓ during the bust on house prices and consumption. The "Total effect of the spike in r_ℓ" column reports the results where the 3.9 percent spike in r_ℓ is fed into the economy at the peak of the boom (all prices are allowed to react, but parameters are fixed). The "r_ℓ direct" column reports the results where all prices except the house prices are fixed. The "wage effect" column reports the results where the wage sequence from the "Total effect of the spike in r_ℓ" exercise is fed into the economy, keeping the interest rate, the return on capital, and the other parameters constant.

percentage points increase during the bust. To isolate the effect of the spike, we run an experiment where at the peak of the boom, the economy is shocked with a 3.9 percentage points increase in the bank lending rate. We solve all prices endogenously.

Table V reports the results of this exercise and compares them with the benchmark results. Despite being temporary, the spike in r_ℓ causes a sizable downturn. Quantitatively, the bank balance sheet deterioration generates a 10.1 percent decline in wages accounting for a big portion of the benchmark decline and a 9.9 percent decline in house prices, and a 6.3 percent decline in consumption. The spike directly lowers house prices by 4.7 percent and consumption by 1 percent. Indirectly via wages, it lowers house prices by 3.6 percent and consumption by 5.3 percent.[35] These results imply that the

[35]We run an alternative experiment where at the peak of the boom, the bank net worth is hit with the bank balance sheet losses of the benchmark economy, keeping all parameters fixed at their boom level (see Table C.1 in Appendix C). Since the bank leverage parameter is fixed at the boom level, banks supply more credit in this experiment than the benchmark despite the same decline in bank net worth. As a result, the spike in bank lending rate r_ℓ in this experiment is 2.1, smaller than the benchmark spike of 3.9 percentage points. Quantitatively, the decline in bank net worth generates 5.6, 5.2, and 3.9 percent declines in wages, house prices, and consumption, respectively, smaller than those generated by the

bank balance sheet deterioration amplifies the bust in variables that depend on short-term debt, such as wages, compared with those variables that depend on long-term debt, such as house prices. Moreover, the direct and indirect effects of the bank lending rate on house prices are comparable, and the indirect effect is significantly more important for the decline in consumption.

5.2.3 Direct and Indirect Effects of Housing Demand and House Price Shocks

The previous literature typically focused on how declines in house prices affect household consumption *directly* through wealth and collateral effects.[36] This household balance sheet mechanism is also present in our model. However, there is an additional *indirect* effect of house price declines in our model: increases in foreclosures lower bank net worth and hence credit supply, causing a spike in the bank lending rate r_ℓ. In this section, we decompose the effects of shocks to the housing market into their direct and indirect effects.[37]

First, we focus on the housing demand shock analyzed in Section 5.1 and study its effects on house prices and consumption during the bust.[38] To find the total (direct and indirect effects combined) effect of the housing demand shock, we shock the economy at the peak of the benchmark boom with only the housing demand shock while keeping the other parameters constant at their peak levels. All prices are determined endogenously in equilibrium. The first column in Table VI (which is the replica of the last column in Table III) reports the results. The second column in Table VI reports the direct effect of the housing demand shock (we close its indirect effect by shutting down the equilibrium response of all prices except for house prices).[39] We find that the

---

benchmark spike but are still sizable. Thus, regardless of the type of exercise, we find that bank balance sheet deterioration has significant effects on the bust.

[36] See Mian et al. (2013), Mian and Sufi (2014), Gertler and Gilchrist (2018), and Berger et al. (2018).

[37] We also use the direct and indirect terminology for the effect of the bank lending rate. In that case, the direct effect is via household borrowing costs and the indirect effect is via firm borrowing costs.

[38] The indirect effect of the housing demand shock during the boom is negligible.

[39] Shutting down the response of the bank lending rate r_ℓ only and solving r_k and w

Table VI – Effects of Housing Demand Shock (HDS) and House Price Shock (P_h-shock)

	HDS bust (total)	HDS direct effect	HDS indirect effect through r_ℓ: r_ℓ effect	r_ℓ effect decomposed: r_ℓ direct	r_ℓ effect decomposed: wage effect
	(i)	(ii)	(iii)	(iv)	(v)
Bank lending rate (Δr_ℓ)	3.2	–	3.2	3.2	–
Wages ($\%\Delta w$)	-8.4	–	-8.4	–	-8.4
House price ($\%\Delta p_h$)	-30.9	-20.0	-8.6	-4.2	-3.1
Consumption ($\%\Delta C$)	-10.1	-3.8	-5.6	-1.0	-4.7

	Exogenous P_h-shock bust (total)	P_h-shock direct effect	P_h-shock indirect effect through r_ℓ: r_ℓ effect	r_ℓ effect decomposed: r_ℓ direct	r_ℓ effect decomposed: wage effect
House price ($\Delta P_h\%$)	-31.9	-31.9	–	–	–
Bank lending rate (Δr_ℓ)	1.6	–	1.6	1.6	–
Wages ($\%\Delta w$)	-3.9	–	-3.9	–	-3.9
Consumption ($\%\Delta C$)	-9.6	-7.4	-1.9	-0.1	-2.0

Notes: This table reports the results of the effects of housing demand and housing price shocks during the bust. "HDS" refers to the housing demand shock. See the text for details.

housing demand shock directly causes a 20 percent decline in house prices and a 3.8 percent decline in consumption.

Next, to quantify the indirect effect of the housing demand shock via bank balance sheets, we shock the economy at the peak of the boom with the r_ℓ sequence obtained from the "HDS bust (total)" experiment and solve for w and r_k endogenously.[40] The rise in r_ℓ reflects the extent of damage the decline in house prices causes on bank balance sheets. The third column reports the results: the rise in r_ℓ causes an 8.6 percent decline in house prices, accounting for 28 percent, and a 5.6 percent decline in consumption, accounting for 55

endogenously has a minimal effect on results since the changes in r_ℓ are the key driving force behind the indirect effect of house prices.

[40]Whether we solve w and r_k endogenously in this experiment or feed into the economy the w and r_k sequence of the HDS bust, we obtain very similar results. The same comment applies to the P_h-shock experiment that we analyze next.

percent of the total effect of the shock. Thus, even though the direct effect of the housing demand bust is more important for the decline in house prices, the indirect effect is also substantial. For consumption, the indirect effect is larger than the direct effect (5.6 percent versus 3.8 percent). We further decompose the indirect effect through r_ℓ into r_ℓ's direct effect versus its indirect effect through wages and find that the direct and indirect effects on house prices are comparable (4.2 percent versus 3.1 percent), whereas the indirect effect on consumption is higher than the direct effect (4.7 percent versus 1.0 percent).

In the second experiment, the economy at the peak of the benchmark boom is unexpectedly shocked with the benchmark house price sequence from 2008 onward (a 31.9 percent decline in house prices, followed by a slow recovery, as in Figure 3), keeping all parameters at their boom levels. We decompose the effects of this shock on consumption by solving all prices endogenously except for the house price since it is given as an exogenous shock. The "Exogenous P_h-shock bust (total)" column in Table VI shows that the equilibrium bank lending rate increases by 1.6 percentage points and wages decline by 3.9 percent. Consumption declines by 9.6 percent. To isolate the direct effect of house prices, we run a version of the model where we feed the house price shock into the model but keep all the other prices and model parameters at the boom level. This exercise generates a 7.4 percent decline in consumption. The implied elasticity of consumption to house prices is 0.23, which is consistent with the elasticity found in Berger et al. (2018).

To isolate the indirect effect of the house price decline on consumption via bank balance sheets, we keep house prices at their boom sequence but feed into the economy the r_ℓ sequence arising from the house price shock and solve for w and r_k endogenously. This r_ℓ shock generates a 1.9 percent decline in consumption, which shows that the indirect effect of house price shock on consumption is sizable. Finally, when we decompose the indirect effect of house prices through r_ℓ into r_ℓ's direct and indirect effects, we find that the indirect effect of house prices transmits to consumption mostly via its effect on wages, a theme that has been recurring consistently throughout all of our decomposition exercises.

We conclude this section with two remarks. First, note that the indirect effect of the housing demand shock on consumption is much larger than that of the P_h-shock (5.6 percent decline versus 1.9 percent decline) even though both shocks generate house price declines of similar magnitudes (30.9 percent versus 31.9 percent). This is because the declines in homeownership demand and house prices together make default a more attractive option than the decline in house prices only. As a result, foreclosures increase twice as much under the housing demand shock as under the P_h-shock. Second, the indirect effect of a housing bust is highly non-linear in the size of the bust. Since households hold some equity in their houses, small declines in house prices do not generate enough increase in foreclosures to hurt bank balance sheets. However, as larger declines in house prices push households into negative equity, the foreclosure rate increases at a disproportionately higher rate, strengthening the bank balance sheet mechanism. We confirm this by generating a housing demand bust with a 6 percent house price decline as opposed to 30.9 percent in the first experiment above. In this case, the bank balance sheet deterioration has almost no contribution to the house price decline and accounts for 29 percent of the consumption decline, which is smaller than the 55 percent found in the first experiment.

5.3 Robustness

We check the robustness of our results to alternative parameterizations of the rental market segmentation parameter (η), the elasticity of substitution between consumption and housing ($1/\epsilon$), the coefficient of risk aversion (σ), the bank leverage level $\widehat{\lambda}$, and the labor utilization function curvature (ψ), which are externally set in our benchmark. As in our benchmark, we recalibrate our model for each alternative exogenously set parameterization and analyze the boom-bust dynamics. For different values of $1/\epsilon$, η, and σ, the model generates boom-bust dynamics that are very similar to the benchmark (Figure D.3 in Appendix D).

The model generates different dynamics for different values of ψ and $\widehat{\lambda}$. Hence, we repeat our decomposition exercises (see Table D.2 in Appendix D for details). Overall, our substantive conclusions do not change. For example, the leverage shock accounts for 56 percent (=17.9/31.9) of the house price decline during the bust in the benchmark economy. Across different values of ψ and $\widehat{\lambda}$, we find that its lowest contribution is 46 percent (=14.5/31.5).

6 Conclusion

In this paper, we developed a heterogeneous-agent model that features interactions between household, firm, and bank balance sheets and is consistent with important cross-sectional facts as well as the dynamics of key aggregate variables around the 2008 boom-bust cycle in the US. We used the model to study the contributions of several factors to the boom-bust. While shocks to housing demand are relatively more important for the house price boom-bust, shocks to bank leverage also contribute significantly to house prices and matter more for the dynamics of output, wages, and consumption. Second, we have shown that the feedback from the bank balance sheets into household and firm borrowing rates, the latter affecting household labor income, plays an important role in the amplification of shocks. We have found that the effects of housing market busts are significantly amplified by the deterioration of bank balance sheets, however, this indirect effect of the housing demand shock is highly nonlinear. For smaller declines in house prices, its importance diminishes because foreclosures do not increase significantly enough to hurt bank balance sheets. Overall, our results show that the change in credit supply—whether it is due to the exogenous shocks to bank leverage or an endogenous response to the bank balance sheet deterioration during the bust—is an important contributor to the boom-bust in the housing market and overall economy.

To ensure the clarity of our analysis, we have omitted several factors that might be relevant for the quantitative results. Notably, we have not integrated monetary policy responses to the crises, as their effectiveness hinges heavily on the anticipated trajectory of such policies. Furthermore, we have not considered the feedback from consumption to output as in HANK models (Kaplan et al. (2018)), which could potentially increase the feedback from house prices to output. We intend to explore these extensions in our future research.

References

Adam, Klaus, Pei Kuang, and Albert Marcet, "House price booms and the current account," *NBER Macroeconomics Annual*, 2012, *26* (1), 77–122.

Adelino, Manuel, Antoinette Schoar, and Felipe Severino, "Loan Originations and Defaults in the Mortgage Crisis: The Role of the Middle Class," *The Review of Financial Studies*, 2016, *29* (7), 1635–1670.

Adrian, Tobias and Hyun Song Shin, "Liquidity and Leverage," *Journal of Financial Intermediation*, 2010, *19* (3), 418–437.

_ , Paolo Colla, and Hyun Song Shin, "Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009," *NBER Macroeconomics Annual*, 2013, *27* (1), 159–214.

Aguiar, Mark and Erik Hurst, "Deconstructing Lifecycle Expenditure," *Journal of Political Economy*, 2013, *121* (3), 437–492.

Aiyar, Shekhar, Charles W Calomiris, John Hooley, Yevgeniya Korniyenko, and Tomasz Wieladek, "The international transmission of bank capital requirements: Evidence from the UK," *Journal of Financial Economics*, 2014, *113* (3), 368–382.

Albanesi, Stefania, Giacomo DeGiorgi, and Jaromir Nosal, "Credit Growth and the Financial Crisis: A New Narrative," Working Paper 23740, National Bureau of Economic Research, Inc 2017.

Arslan, Yavuz, Bulent Guler, and Temel Taskin, "Joint Dynamics of House Prices and Foreclosures," *Journal of Money, Credit and Banking*, 2015, *47* (S1), 133–169.

Begenau, Juliane, Saki Bigio, and Jeremy Majerovitz, "Data Lessons on Bank Behavior," *Manuscript*, 2018.

_ , _ , _ , and Matias Vieyra, "Banks Adjust Slowly: Evidence and Lessons for Modeling," *Available at SSRN 3394601*, 2019.

Benmelech, Efraim, Carola Frydman, and Dimitris Papanikolaou, "Financial frictions and employment during the great depression," *Journal of Financial Economics*, 2019, *133* (3), 541–563.

Berger, David, Veronica Guerrieri, Guido Lorenzoni, and Joseph Vavra, "House Prices and Consumer Spending," *The Review of Economic Studies*, 2018, *85* (3), 1502–1542.

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, March 1989, *79* (1), 14–31.

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist, "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1999, *1*, 1341–1393.

Bianchi, Javier and Saki Bigio, "Banks, liquidity management and monetary policy," Technical Report, National Bureau of Economic Research 2014.

Buera, Francisco J and Benjamin Moll, "Aggregate implications of a credit crunch: The importance of heterogeneity," *American Economic Journal: Macroeconomics*, 2015, *7* (3), 1–42.

Campbell, John Y., Stefano Giglio, and Parag Pathak, "Forced Sales and House Prices," *American Economic Review*, 2011, *101* (5), 2108–2131.

Chatterjee, Satyajit and Burcu Eyigungor, "A Quantitative Analysis of the US Housing and Mortgage Markets and the Foreclosure Crisis," *Review of Economic Dynamics*, 2015, *18* (2), 165–184.

_ , Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull, "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, November 2007, *75* (6), 1525–1589.

Chernenko, Sergey, Samuel G Hanson, and Adi Sunderam, "The Rise and Fall of Demand for Securitizations," Working Paper 20777, National Bureau of Economic Research, Inc 2014.

Chodorow-Reich, Gabriel, "The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis," *The Quarterly Journal of Economics*, 2013, *129* (1), 1–59.

Coimbra, Nuno and Hélene Rey, "Financial Cycles with Heterogeneous Intermediaries," Working Paper 23245, NBER 2017.

Committee on the Global Financial System, "The Role of Margin Requirements and Haircuts in Procyclicality," *CGFS Papers No 36*, 2010.

Corbae, Dean and Erwan Quintin, "Leverage and the Foreclosure Crisis," *Journal of Political Economy*, 2015, *123* (1), 1–65.

_ and Pablo D'Erasmo, "A quantitative model of banking industry dynamics," *Federal Reserve Bank of Philadelphia Working Paper*, 2013, (14-13), 14.

_ and _ , "Capital requirements in a quantitative model of banking industry dynamics," Technical Report, National Bureau of Economic Research 2019.

Diamond, William and Tim Landvoigt, "Credit cycles with market-based household leverage," *Journal of Financial Economics*, 2022, *146*, 726–753.

Elenev, Vadim, "Mortgage credit, aggregate demand, and unconventional monetary policy," *Working paper*, 2017.

_ , Tim Landvoigt, and Stijn Van Nieuwerburgh, "Phasing out the GSEs," *Journal of Monetary Economics*, 2016, *81*, 111–132.

_ , _ , and _ , "A macroeconomic model with financially constrained producers and intermediaries," Technical Report, National Bureau of Economic Research 2018.

Favara, Giovanni and Jean Imbs, "Credit Supply and the Price of Housing," *American Economic Review*, 2015, *105* (3), 958–992.

Favilukis, Jack, Sydney C. Ludvigson, and Stijn Van Nieuwerburgh, "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, 2017, *125* (1), 140–223.

Federal Reserve Bank of New York, "Quarterly Trends for Consolidated U.S. Banking Organizations: First Quarter 2020," 2020.

Federal Reserve Board, https://www.federalreserve.gov/pubs/refinancings/ August 2008.

_ , "Financial Stability Report," 2019.

Foote, Christopher L., Lara Loewenstein, and Paul S. Willen, "Cross-Sectional Patterns of Mortgage Debt during the Housing Boom: Evidence and Implications," Working Paper 22985, NBER 2016.

Fraisse, Henri, Mathias Lé, and David Thesmar, "The real effects of bank capital requirements," *Management Science*, 2020, *66* (1), 5–23.

Ganong, Peter and Pascal Noel, "Liquidity versus wealth in household debt obligations: Evidence from housing policy in the great recession," *American Economic Review*, 2020, *110* (10), 3100–3138.

Gertler, Mark and Nobuhiro Kiyotaki, "Financial Intermediation and Credit Policy in Business Cycle Analysis," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Elsevier, 2010, pp. 547–599.

_ and _ , "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, 2015, *105* (7), 2011–2043.

_ and Peter Karadi, "A model of unconventional monetary policy," *Journal of Monetary Economics*, 2011, *58*, 17–34.

— **and Simon Gilchrist**, "What happened: Financial factors in the great recession," *Journal of Economic Perspectives*, 2018, *32* (3), 3–30.

Gete, Pedro and Franco Zecchetto, "Mortgage design and slow recoveries: The role of recourse and default," *Review of Economic Studies*, 2024, *91* (2), 1039–1084.

— **and Michael Reher**, "Mortgage supply and housing rents," *The Review of Financial Studies*, 2018, *31* (12), 4884–4911.

Gilchrist, Simon and Egon Zakrajšek, "Credit spreads and business cycle fluctuations," *American Economic Review*, 2012, *102* (4), 1692–1720.

Glaeser, Edward L, Joshua D Gottlieb, and Joseph Gyourko, "Can cheap credit explain the housing boom?," in "Housing and the financial crisis," University of Chicago Press, 2012, pp. 301–359.

Greenwald, Daniel, "The Mortgage Credit Channel of Macroeconomic Transmission," 2016 Meeting Papers 1551, Society for Economic Dynamics 2016.

Gropp, Reint, Thomas Mosk, Steven Ongena, and Carlo Wix, "Banks response to higher capital requirements: Evidence from a quasi-natural experiment," *The Review of Financial Studies*, 2019, *32* (1), 266–299.

Guler, Bulent, "Innovations in Information Technology and the Mortgage Market," *Review of Economic Dynamics*, 2015, *18* (3), 456–483.

Guvenen, Fatih and Anthony A Smith, "Inferring Labor Income Risk and Partial Insurance from Economic Choices," *Econometrica*, November 2014, *82* (6), 2085–2129.

Harding, John, Stuart Rosenthal, and Sirmans C.F., "Depreciation of housing capital, maintenance, and house price inflation: Estimates from a repeat sales model," *Journal of Urban Economics*, 2007, *61* (2), 193–217.

Hatchondo, Juan Carlos, Leonardo Martinez, and Juan M. Sanchez, "Mortgage Defaults," *Journal of Monetary Economics*, 2015, *76*, 173–190.

Huo, Zhen and Jose-Victor Rios-Rull, "Balance Sheet Recessions," Working Papers, Federal Reserve Bank of Minneapolis 2013.

Ivashina, Victoria and David Scharfstein, "Bank lending during the financial crisis of 2008," *Journal of Financial economics*, 2010, *97* (3), 319–338.

Jayaratne, Jith and Philip E Strahan, "The benefits of branching deregulation," *Economic Policy Review*, 1997, *3* (4).

Jeske, Karsten, Dirk Krueger, and Kurt Mitman, "Housing, Mortgage Bailout Guarantees and the Macro Economy," *Journal of Monetary Economics*, 2013, *60*

(8), 917–935.

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró, and Jesús Saurina, "Macroprudential policy, countercyclical bank capital buffers, and credit supply: Evidence from the Spanish dynamic provisioning experiments," *Journal of Political Economy*, 2017, *125* (6), 2126–2177.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, "Credit Supply and the Housing Boom," *Journal of Political Economy*, 2017.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary Policy According to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

_ , Kurt Mitman, and Giovanni L Violante, "The housing boom and bust: Model meets evidence," *Journal of Political Economy*, 2020, *128* (9), 3285–3345.

Keys, Benjamin J, Tomasz Piskorski, Amit Seru, and Vikrant Vig, "Mortgage financing in the housing boom and bust," in "Housing and the Financial Crisis," University of Chicago Press, 2012, pp. 143–204.

Kiyotaki, Nobuhiro, Alexander Michaelides, and Kalin Nikolov, "Winners and Losers in Housing Markets," *Journal of Money, Credit and Banking*, 2011, *43*, 255–296.

_ and John Moore, "Credit cycles," *Journal of political economy*, 1997, *105* (2), 211–248.

Landvoigt, Tim, "Financial intermediation, credit risk, and credit supply during the housing boom," *Working paper*, September 2016.

Livshits, Igor, James MacGee, and Michele Tertilt, "Accounting for the Rise in Consumer Bankruptcies," *American Economic Journal: Macroeconomics*, April 2010, *2* (2), 165–93.

_ , Jim MacGee, and Michele Tertilt, "Consumer Bankruptcy – A Fresh Start," *American Economic Review*, 2007, *97* (1), 402–418.

Marco, Filippo De, Christiane Kneer, and Tomasz Wieladek, "The real effects of capital requirements and monetary policy: Evidence from the United Kingdom," *Journal of Banking & Finance*, 2021, *133*, 106237.

Mendoza, Enrique and Vincenzo Quadrini, "Financial Globalization, Financial Crises and Contagion," *Journal of Monetary Economics*, 2010, *57* (1), 24–39.

Mian, Atif and Amir Sufi, "What Explains the 2007-2009 Drop in Employment?," *Econometrica*, 2014, *82* (6), 2197–2223.

_ , Kamalesh Rao, and Amir Sufi, "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 2013, *128* (4), 1687–1726.

Mitman, Kurt, "Macroeconomic Effects of Bankruptcy and Foreclosure Policies," *American Economic Review*, 2016, *106* (8), 2219–2255.

Nuno, Galo and Carlos Thomas, "Bank Leverage Cycles," *American Economic Journal: Macroeconomics*, 2017, *9* (2), 32–72.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, "Housing, consumption and asset pricing," *Journal of Financial Economics*, March 2007, *83* (3), 531–569.

Popov, Alexander A and Jörg Rocholl, "Financing constraints, employment, and labor compensation: evidence from the subprime mortgage crisis," 2015.

Sherman, Matthew, "A Short History of Financial Deregulation in the United States," CEPR Discussion Papers, C.E.P.R. Discussion Papers 2009.

Shin, Hyun Song, "Securitisation and Financial Stability," *The Economic Journal*, 2009, *119* (536), 309–332.

Sommer, Kamila, Paul Sullivan, and Randal Verbrugge, "The equilibrium effect of fundamentals on house prices and rents," *Journal of Monetary Economics*, 2013, *60* (7), 854–870.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, June 2004, *112* (3), 695–717.

ONLINE APPENDIX

A Data

GDP Real gross domestic product per capita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Aggregate Consumption Real personal consumption expenditures per capita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Labor income Total wages and salaries (Not Seasonally Adjusted Annual Rate from FRED) divided by working-age population and then divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use annual data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2018 from this trend.

Hours per person Hours of Wage and Salary Workers on Nonfarm Payrolls (from FRED, Total, Billions of Hours, Quarterly, Seasonally Adjusted Annual Rate) divided by Working Age Population (from FRED, Aged 15-64: All Persons for the United States, Persons, Quarterly, Seasonally Adjusted).

Investment Private Nonresidential Fixed Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Homeownership Rate Census Bureau Homeownership rate for the U.S. (Table 14) and by age of the householder (Table 19). Housing Vacancies and Homeownership (CPS/HVS) - Historical Tables.

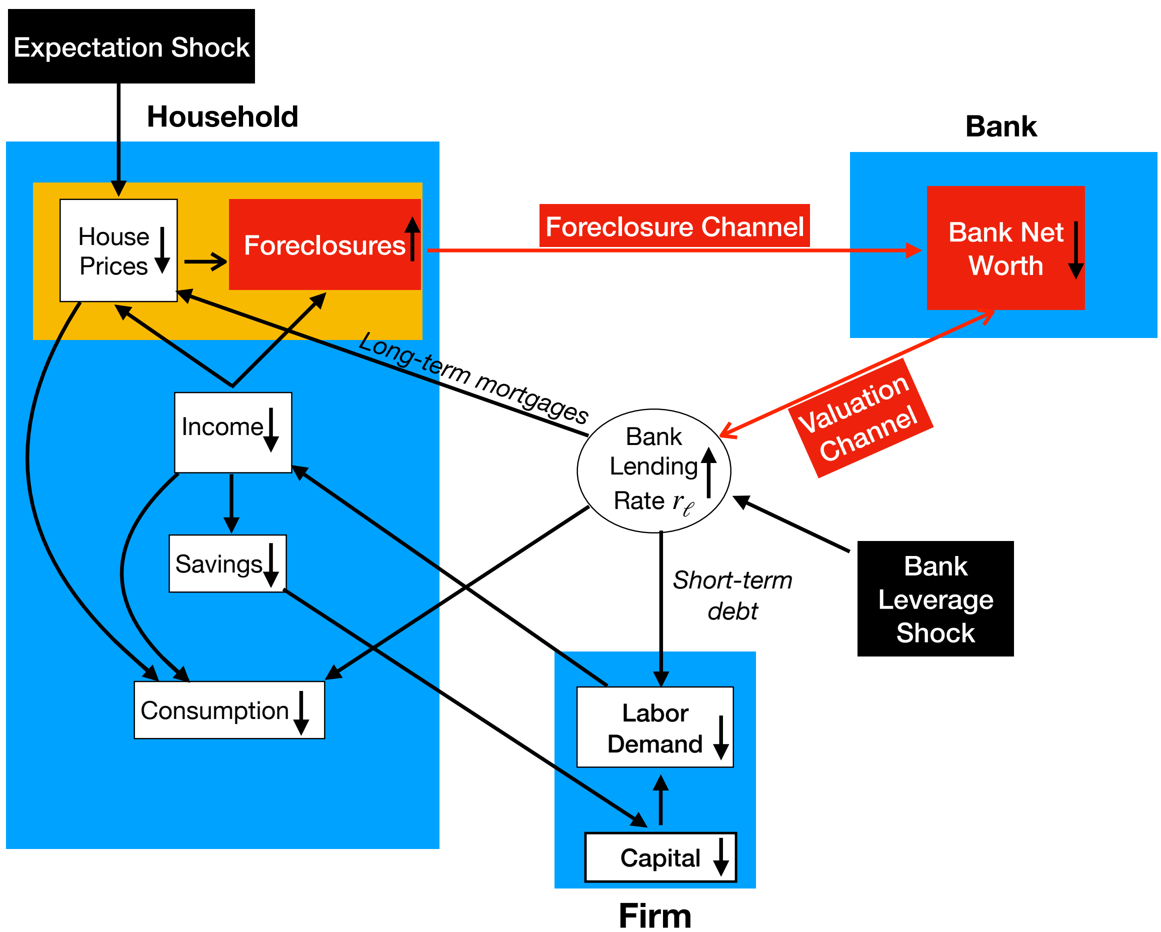
House Prices House Price Index for the entire US (Source: Federal Housing Finance Agency) divided by the price index for nondurable consumption (line 6 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percent deviation of the data from 1985 to 2018 from this trend. To obtain the changes relative to GDP, we divide the real house price index by the real GDP series.

Mortgage Debt and Household Leverage Home Mortgage Liabilities and Mortgage Liabilities divided by Owner Occupied Housing Real Estate at Market Value. Source: Flow of Funds B.101 Balance Sheet of Households and Nonprofit Organizations.

B Extra Figures

B.1 Linkages across sectors

FIGURE B.1 – Linkages across sectors and amplification channels during the bust



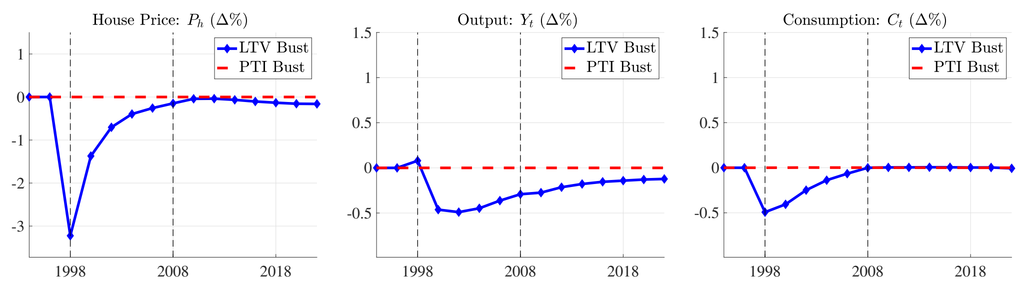
C Extensions

TABLE C.1 – Effects of Bank Net Worth Shock during the Bust

Variables	Benchmark	Bank net worth shock
Bank lending rate (Δr_ℓ)	4.4	2.1
Wages ($\%\Delta w$)	-11.5	-5.6
House Prices ($\%\Delta p_h$)	-31.9	-5.2
Consumption ($\%\Delta C$)	-11.5	-3.9

Notes: This table reports the results of the exercise where we shock the economy at the peak of the boom with bank net worth losses from the benchmark economy during the bust. We solve for all prices endogenously. The parameters remain at the peak of the boom values.

FIGURE C.2 – LTV and PTI Busts



Notes: This figure plots the model dynamics where LTV and PTI constraints tighten unexpectedly. The LTV constraint tightens from 100 to 80 percent, and the PTI constraint tightens to 25 percent (no PTI constraints in the benchmark) with a persistence of 0.6. We focus on the bust since relaxation of the constraints does not generate significant booms.

D Robustness

For all robustness analysis, we recalibrate the initial steady state of the model. Then as a first step, we feed into the economy the same boom-bust shocks from our benchmark. For the boom period, if the model generates a boom very close to the benchmark, we do not recalibrate the boom. If the boom in housing prices and bank leverage is different, then we recalibrate the housing demand shock and the leverage shock to match the size of the housing boom and the increase in bank leverage. The boom-bust dynamics look almost identical for different values of η, ϵ, and σ as shown in Figure D.3.

TABLE D.2 – Effects of Shocks to Leverage (LS) and Housing Demand (HDS) under Different Parameterizations

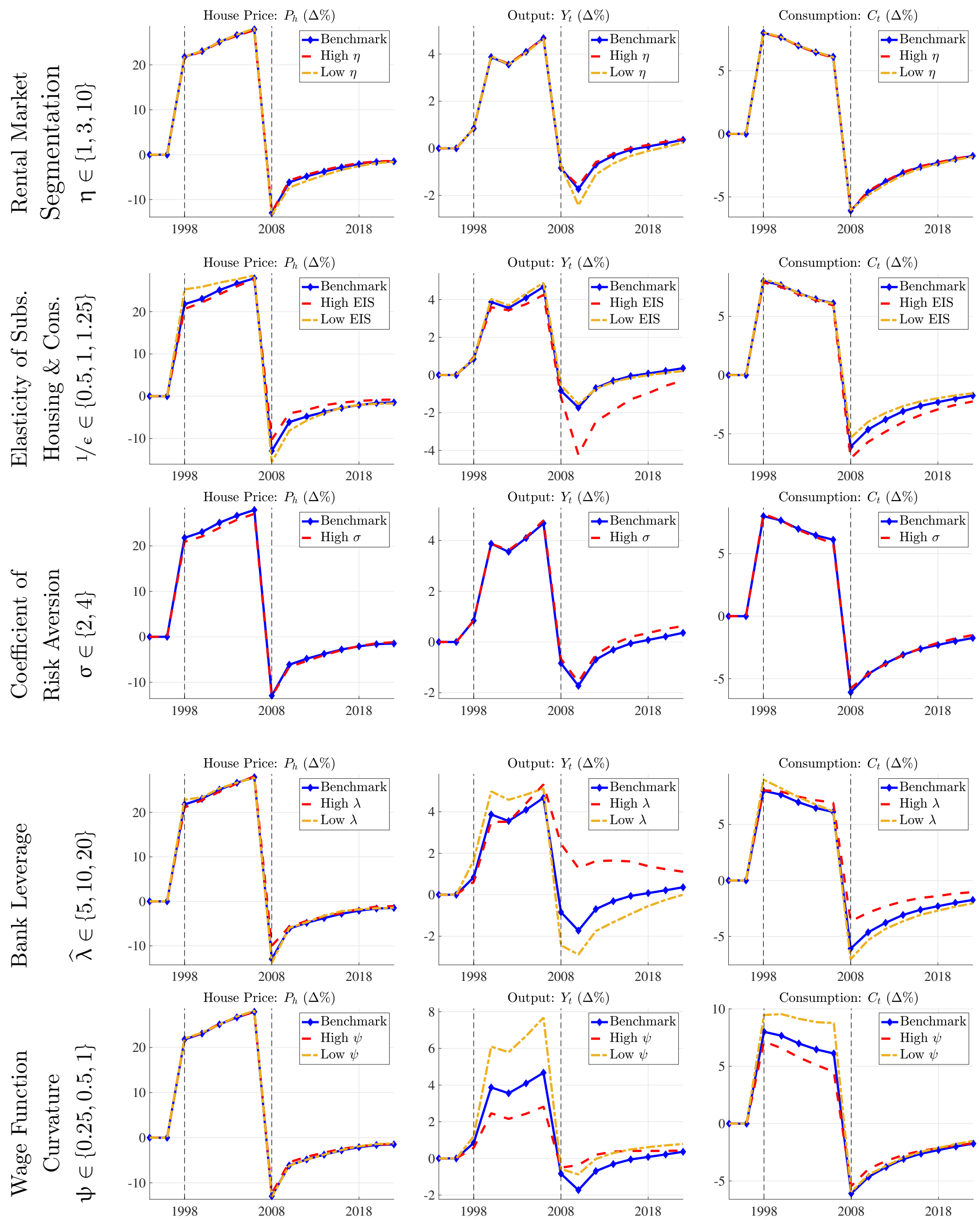
Benchmark $\psi = 0.5$ & $\widehat{\lambda} = 10$	Two-shock Bust	LS Bust	HDS Bust	HDS direct effect	HDS Indirect effect through r_ℓ
House price ($\%\Delta p_h$)	-31.9	-17.9	-30.9	-20.0	-8.6
Consumption ($\%\Delta C$)	-11.5	-9.9	-10.1	-3.8	-5.6
$\psi = 0.25$					
House price ($\%\Delta p_h$)	-32.2	-22.6	-22.9	-11.1	-5.1
Consumption ($\%\Delta C$)	-13.5	-12.9	-7.5	-1.2	-4.1
$\psi = 1$					
House price ($\%\Delta p_h$)	-31.5	-14.5	-30.9	-22.1	-6.9
Consumption ($\%\Delta C$)	-9.5	-7.4	-8.5	-3.7	-4.0
$\widehat{\lambda} = 20$					
House price ($\%\Delta p_h$)	-29.8	-16.4	-20.6	-11.9	-2.5
Consumption ($\%\Delta C$)	-9.9	-8.5	-5.7	-1.5	-2.4
$\widehat{\lambda} = 5$					
House price ($\%\Delta p_h$)	-32.5	-20.0	-30.9	-15.1	-8.0
Consumption ($\%\Delta C$)	-12.4	-11.3	-9.6	-2.0	-5.1

Notes: This table reports robustness results for the bust period for different parameter values. "LS" refers to the leverage shock and "HDS" refers to the housing demand shock. See the text for details.

For parameter variations that generate a difference, we further investigate by calibrating the the boom-bust dynamics as in our benchmark model. The response of hours (hence, of output and wages) to changes in r_ℓ depends on the labor utilization curvature ψ. We set $\psi = 0.5$ so that the employment response to changes in credit spreads in our benchmark is consistent with the data. For higher values of ψ, the response of employment to changes in r_ℓ becomes smaller; hence, the importance of the changes in credit supply (both exogenous and endogenous) for the boom-bust. To check the sensitivity of our results, we conduct our decomposition exercises with $\psi \in \{0.25, 1\}$ (Table D.2). When $\psi = 1$, the leverage shock generates declines of 14.5 and 7.4 percent in house prices and consumption, respectively, as opposed to 17.9 and 9.9 in our benchmark. The indirect effects of the housing demand shock on house prices and consumption go down to 6.9 percent and 4.0 percent, respectively, from 8.6 percent and 5.6 percent in our benchmark. When $\psi = 0.25$, the effect of the leverage shock becomes larger. As a result, the size of the housing demand shock necessary to match the boom gets smaller, which lowers both the direct and the indirect effects of the housing demand shock. Overall, the leverage shock can generate 46, 56, and 70 percent of the benchmark house price declines for $\psi = 0.25, 0.5$, and 1, respectively. In either case, we conclude that the leverage shock significantly contributes to the bust in house prices. We reach a similar conclusion when we focus on other decomposition exercises.

The robustness with respect to bank leverage shows that the bank leverage level also influences the model dynamics. To check the sensitivity of our results, we conduct our decomposition exercises with $\widehat{\lambda} \in \{5, 20\}$. As bank leverage increases, the effect of the leverage shock declines. Even if $\widehat{\lambda} = 20$, where the effect of the bank leverage shock is smallest, the leverage shock can generate 16.4/29.8=55 percent of the benchmark decline in house prices as opposed to 17.9/31.9=56 percent in our benchmark (with $\widehat{\lambda} = 10$). For $\widehat{\lambda} = 5$, the corresponding number is 62 percent. Overall, as can be seen, the contribution of the leverage shock to the bust does not vary significantly. The indirect effect of the housing demand shock has a non-monotonic pattern with respect to $\widehat{\lambda}$. For $\widehat{\lambda} = 20$, the overall effect of the housing demand shock is smaller (20.6 percent decline in house prices as opposed to 30.9 in our benchmark). As a result, it generates a smaller amplification from endogenous credit supply changes (2.5 percent decline in house prices as opposed to the 9.6 percent in the benchmark). However, notice also that the direct effect of the housing demand shock is also smaller: 11.9 percent as opposed to 20 percent in the benchmark.

Figure D.3 – The two-shock boom-bust dynamics with alternative parameterizations



Notes: This figure plots robustness analysis with respect to several model parameters.

E Value Functions for Households

E.1 Active Renters

An active renter has two choices: continue to rent or purchase a house, that is, $V^r = \max\left\{V^{rr}, V^{rh}\right\}$ where V^{rr} is the value function if she decides to continue renting and V^{rh} is the value function if she decides to purchase a house. If she decides to continue to rent, she chooses rental unit size s at price p_r per unit, makes her consumption and saving choices, and remains as an active renter in the next period. After purchasing a house, she begins the next period as a homeowner. The value function of an active renter who decides to remain as a renter is given by

$$V_j^{rr}(a, z) = \max_{c,s,a' \geqslant 0} \left\{u(c, s) + \beta E V_{j+1}^r(a', z')\right\} \tag{E.1}$$

subject to

$$c + \frac{a'}{1+r} + p_r s = y(j, z) + a,$$

where a is the beginning-of-period financial wealth, $p_r s$ is the rental payment, r_i is the return to savings, and w is the wage rate per efficiency unit of labor. The expectation operator is over the income shock z'.

E.2 Inactive Renters

Inactive renters are not allowed to purchase a house because of their default in previous periods. However, they can become active renters with probability π. Since they cannot buy a house; they only make rental size, consumption, and saving decisions. The value function of an inactive renter is given by

$$V_j^e(a, z) = \max_{c,s,a' \geqslant 0} \left\{u(c, s) + \beta\left[\pi E V_{j+1}^r(a', z') + (1-\pi) E V_{j+1}^e(a', z')\right]\right\} \tag{E.2}$$

subject to

$$c + \frac{a'}{1+r} + p_r s \quad = \quad y(j, z) + a.$$

E.3 Homeowners

The options of a homeowner are: 1) stay as a homeowner, 2) refinance, 3) sell the current house (become a renter or buy a new house), or 4) default. The value function of an owner is given as the maximum of these four options, that is, $V^h = \max\left\{V^{hh}, V^{hf}, V^{hr}, V^{he}\right\}$, where V^{hh} is the value of staying as a homeowner, V^{hf} is the value of refinancing, V^{hr} is the value of selling, and V^{he} is the value of defaulting (being excluded from the ownership option).

A stayer makes a consumption and saving decision given his income shock, housing, mortgage debt, and assets. Therefore, the problem of the stayer can be formulated as follows:

$$V_j^{hh}(a,h,d,z) = \max_{c,a'\geqslant 0} \left\{u(c,h) + \beta E V_{j+1}^{h}\left(a',h,d',z'\right)\right\} \tag{E.3}$$

subject to

$$\begin{aligned} c + \delta_h p_h h + \frac{a'}{1+r} + m &= y(j,z) + a \\ d' &= (d-m)(1+r_\ell), \end{aligned}$$

where m is the mortgage payment following the standard amortization schedule computed at the bank lending rate r_ℓ.

The second choice for the homeowner is to refinance, which also includes prepayment. Refinancing requires paying the full balance of any existing debt and getting a new mortgage. We assume that refinancing is subject to the same transaction costs as new mortgage originations. So, we can formulate the problem of a refinancer as

$$V_j^{hf}(a,h,d,z) = \max_{c,d',a'\geqslant 0} \left\{u(c,h) + \beta E V_{j+1}^{h}(a',h,d',z')\right\} \tag{E.4}$$

subject to

$$c + d + \delta_h p_h h + \varphi_f + \frac{a'}{1+r} = y(j,z) + a + d'\left(q^m(d';a,h,z,j) - \varphi_m\right).$$

The third choice for the homeowner is to sell the current house and either stay as a renter or buy a new house. Selling a house is subject to a transaction cost that equals fraction φ_s of the selling price. Moreover, a seller has to pay the outstanding mortgage debt, d, in full to the lender. A seller, upon selling the house, can either rent a house or buy a new one. Her problem is identical to a renter's problem. So, we have

$$V_j^{hr}(a,h,d,z) = V_j^{r}\left(a + p_h h(1-\varphi_s) - d, z\right).$$

The fourth possible choice for a homeowner is to default on the mortgage, if she has one. A defaulter has no obligation to the bank. The bank seizes the house, sells it on the market, and returns any positive amount from the sale of the house, net of the outstanding mortgage debt and transaction costs, back to the defaulter. For the lender, the sale price of the house is assumed to be $(1-\varphi_e)\,p_h h$. Therefore, the defaulter receives $\max\{(1-\varphi_e)\,p_h h - d, 0\}$ from the lender. The defaulter starts the next period as an active renter with probability π. With probability $(1-\pi)$, she stays as an inactive renter. The problem of a defaulter becomes the following:

$$V_j^{he}(a,d,z) = \max_{c,s,a'\geqslant 0} \left\{u(c,s) + \beta E\left[\pi V_{j+1}^{r}\left(a',z'\right) + (1-\pi)\,V_{j+1}^{e}\left(a',z'\right)\right]\right\} \tag{E.5}$$

subject to

$$c+\frac{a'}{1+r}+p_r s \quad = \quad a+y\,(j,z)+\max\{(1-\varphi_e)\,p_h h-d,0\}\,.$$

The problem of a defaulter is different from the problem of a seller in two ways. First, the defaulter receives $\max\{(1-\varphi_e)\,p_h h-d,0\}$ from the housing transaction, whereas a seller receives $(1-\varphi_s)\,p_h h-d$. We assume that the default cost is higher than the sale transaction cost, that is, $\varphi_e>\varphi_s$, and the defaulter receives less than the seller as long as $(1-\varphi_s)\,p_h h-d\geqslant 0$ (i.e., the home equity net of the transaction costs for the homeowner is positive). Second, a defaulter does not have access to the mortgage in the next period with some probability. Such an exclusion lowers the continuation utility for a defaulter. In sum, since defaulting is costly, a homeowner will choose to sell the house instead of defaulting as long as $(1-\varphi_s)\,p_h h-d\geqslant 0$ (i.e., net home equity is positive). Hence, negative equity is a necessary condition for default in the model. Therefore, in equilibrium, a defaulter gets nothing from the lender.

F Firm's Problem

The firm's first-order conditions are given as

$$\begin{aligned}
\alpha\mathbb{Z}_t\left(\frac{K_t}{N_t u_t}\right)^{\alpha-1} &= r_{k,t}+\delta \\
(1-\alpha)\,\mathbb{Z}_t u_t\left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= \left(1+\mu r_{\ell,t+1}\right)\left(\bar{w}_t+\vartheta\frac{u_t^{1+\psi}}{1+\psi}\right) \\
(1-\alpha)\,\mathbb{Z}_t\left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= \left(1+\mu r_{\ell,t+1}\right)\vartheta u_t^{\psi}.
\end{aligned}$$

G Rental Companies

The objective of the company is to maximize its total market value:

$$V_t^{rc}\left(H_{t-1}^r\right)=\max_{H_t^r}\frac{p_t^h\,(1-\delta)\,H_{t-1}^r-p_t^h H_t^r-\frac{\eta p_t^h\left(H_t^r-H_{t-1}^r\right)^2}{2}+\left(p_t^r-\kappa\right)H_t^r+V_{t+1}^{rc}\left(H_t^r\right)}{1+r_{k,t}},$$

which leads to the following first-order condition:

$$p_t^r=\kappa+p_t^h+\eta p_t^h\left(H_t^r-H_{t-1}^r\right)-\frac{(1-\delta_h)\,p_{t+1}^h+\eta p_{t+1}^h\left(H_{t+1}^r-H_t^r\right)}{1+r_{k,t+1}}. \quad \text{(G.1)}$$

In order to see how p_t^r is affected by p_t^h and the homeownership rate, first consider the case where $\eta=0$, which corresponds to the frictionless housing market explored in Kaplan et al. (2020). Equation (G.1) in this case becomes

$$p_t^r = \kappa + p_t^h - \frac{(1-\delta_h)\, p_{t+1}^h}{1 + r_{k,t+1}}.$$

This equation implies that, for a given p_t^h, a higher future house price p_{t+1}^h reduces p_t^r. This is the main mechanism in Kaplan et al. (2020) that generates an increase in the price-rent ratio. However, the homeownership rate does not have any effect on the rental price in this case. So, policies, such as a relaxation of LTV limits that affect the homeownership rate, do not move the price-rent ratio.

H Symmetric Equilibrium Conditions

Labor market: The labor market clears in all periods(i.e., $N_t = 1$).

Credit market: Let $\Gamma_t(\theta)$ be the distribution of available mortgages after households make their decisions at time t. In equilibrium, 1) $\ell_{t+1}(\theta) = \Gamma_t(\theta)$ (the representative bank holds the equilibrium mortgage portfolio), 2) $L_{t+1} = \mu w(\bar{w}_t, u_t) + \int_\theta p_t(\theta)\Gamma_t(\theta)$, which determines $r_{\ell,t+1}$, and 3) $A_{t+1} = K_{t+1} + V_{t+1}^{rc}(H_t^r)$, which determines $r_{k,t+1}$.

Housing market: Remember that total housing supply is fixed at H. Thus, the total demand of owners and renters should be equal to the supply, which determines house price $p_h(t)$. Given house prices $p_h(t)$ and $p_r(t)$, households solve their optimal housing choices, which gives the demand for owner-occupied units $H_t^{o,D}$ and rental units $H_t^{r,D}$. The supply of rental housing units is given by the first-order condition of the rental company (equation G.1). Then, the following two equilibrium conditions give the house price p_t^h and rental prices p_t^r: $H_t^{r,S} = H_t^{r,D}$ and $\bar{H} = H_t^{r,D} + H_t^{o,D}$.

Government: The government runs a pay-as-you-go pension system. It collects social security taxes from working-age households and distributes to retirees. We assume the pension system runs a balanced budget:

$$\sum_{j=1}^{J_R} \sum_z \tau y(j,z)\, \pi_j(z) = \sum_{j=J_R+1}^{J} \sum_z y_R(j,z)\, \pi_j(z),$$

where $\pi_j(z)$ is the measure of individuals with income shock z at age j.

I Characterization of the Bank's Problem

In this section, we will provide proofs for the characterization of the bank's problem. We will start with the steady-state value functions and decision rules and continue obtaining value functions in the transition by iterating backward from the steady state.

The bank's problem is given as

$$\Psi_t\left(L_t, B_t\right) = \max_{B_{t+1}, L_{t+1}, c_t^B} \left\{\log\left(c_t^B\right) + \beta_L \Psi_{t+1}\left(L_{t+1}, B_{t+1}\right)\right\}$$

subject to

$$\begin{aligned} c_t^B + L_{t+1} &= \left(1 + r_{\ell,t}\right) L_t - \left(1 + r_t\right) B_t + B_{t+1} \\ \Psi_{t+1}\left(L_{t+1}, B_{t+1}\right) &\geqslant \tilde{\Psi}_{t+1}^D\left(\xi\left(1 + r_{\ell,t+1}\right) L_{t+1}\right), \end{aligned}$$

where $\tilde{\Psi}_t^D(W) = \max_{W'} \log\left(W - W'\right) + \beta_L \tilde{\Psi}_{t+1}^D\left((1 + r_{t+1})W'\right)$.

I.1 Steady State with $r_\ell > r$

We will characterize the case $r_\ell > r$ and leave the cases for $r_\ell \leqslant r$ for brevity. We will start with the value function of the bank when it defaults.

Since the bank can steal a fraction ξ of assets after the return has been realized and can continue saving at interest rate r, the bank's problem in the period of default is given as

$$\tilde{\Psi}^D\left(\xi L'\right) = \max_{s'} \log\left(\xi L' - W'\right) + \beta_L \Psi^D\left((1 + r)W'\right),$$

and after default, it becomes

$$\tilde{\Psi}^D(W) = \max_{s'} \log\left(W - W'\right) + \beta_L \Psi^D\left((1 + r)W'\right).$$

Lemma 1. $\tilde{\Psi}^D(W)$ *is given as*

$$\tilde{\Psi}^D(W) = \frac{1}{1 - \beta_L}\log(W) + \frac{\beta_L}{(1 - \beta_L)^2}\log(\beta_L(1 + r)) + \frac{\log(1 - \beta_L)}{1 - \beta_L}.$$

The bank's problem in the no-default state solves

$$\Psi(L, B) = \max_{L', B'} \log\left((1 + r_\ell)L - (1 + r)B + B' - L'\right) + \beta_L \Psi\left(L', B'\right)$$

subject to

$$\Psi\left(L', B'\right) \geqslant \tilde{\Psi}^D\left(\xi(1 + r_\ell)L'\right).$$

Proposition 1. *The solution to the bank's problem is given as follows:*

1. Value function:

$$\begin{aligned}\Psi(L,B) &= \frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right)\\ &+ \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r)(1+r_\ell)\beta_L\phi}{1+r-(1+r_\ell)(1-\phi)}\right)+\frac{\log(1-\beta_L)}{1-\beta_L}.\end{aligned}$$

2. The no-default constraint can be written as

$$(1+r_\ell)(1-\phi)L' \geqslant (1+r)B'$$

where ϕ is given as

$$\phi = \xi^{1-\beta_L}\left(\frac{1+r}{1+r_\ell}-(1-\phi)\right)^{\beta_L}.$$

3. The bank's solution satisfies the following expression regardless of whether or not the no-default constraint binds:

$$L'-B' = \beta_L\left((1+r_\ell)L-(1+r)B\right).$$

4. The decision rules when the no-default constraint is binding (if $r_\ell > r$):

$$\begin{aligned}L' &= \frac{(1+r)}{1+r-(1-\phi)(1+r_\ell)}\beta_L\left((1+r_\ell)L-(1+r)B\right)\\ B' &= \frac{(1-\phi)(1+r_\ell)}{1+r-(1-\phi)(1+r_\ell)}\beta_L\left((1+r_\ell)L-(1+r)B\right).\end{aligned}$$

Proof. (Proposition 1) We will use the expressions for value functions and verify the claims above. First, drive the capital requirement constraint:

$$\Psi\left(L',B'\right) \geqslant \tilde{\Psi}^D\left(\xi(1+r_\ell)L'\right).$$

$$\begin{aligned}\frac{1}{1-\beta_L}\log\left((1+r_\ell)L'-(1+r)B'\right)+\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right) &\geqslant\\ \frac{1}{1-\beta_L}\log(\xi(1+r_\ell)L')+\frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r)),&\end{aligned}$$

where ϕ' is the capital requirement constraint in the next period. The expression above gives

$$\log\left(\frac{(1+r_\ell)L'-(1+r)B'}{\xi(1+r_\ell)L'}\right) \geqslant \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta\left((1+r)-(1+r_\ell)(1-\phi')\right)}{(1+r_\ell)\beta_L\phi'}\right)$$

$$\frac{(1+r_\ell)L' - (1+r)B'}{(1+r_\ell)L'} \geqslant \xi\left(\frac{((1+r)-(1+r_\ell)(1-\varphi'))}{(1+r)\varphi'}\right)^{\frac{\beta_L}{1-\beta_L}}.$$

We will show below that the solution of φ' is the fixed point of

$$\varphi = \xi\left(\frac{((1+r)-(1+r_\ell)(1-\varphi'))}{(1+r)\varphi'}\right)^{\frac{\beta_L}{1-\beta_L}}.$$

Then this constraint can be written as

$$(1+r_\ell)(1-\varphi)L' \geqslant (1+r)B'.$$

Now, we can solve the bank's problem:

$$\begin{aligned}
\Psi(L,B) &= \max_{L',B'} \log\left((1+r_\ell)L - (1+r)B + B' - L'\right) + \beta_L \Psi\left(L', B'\right) \\
&= \max_{L',B'} \log\left((1+r_\ell)L - (1+r)B + B' - L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L' - (1+r)B'\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\varphi'}{(1+r)-(1+r_\ell)(1-\varphi')}\right) + \frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

subject to

$$(1+r_\ell)(1-\varphi)L' \geqslant (1+r)B'.$$

Imposing the balance sheet constraint, we obtain

$$\begin{aligned}
\Psi(L,B) &= \max_{L',B'} \log\left((1+r_\ell)L - (1+r)B + \frac{(1+r_\ell)(1-\varphi)L'}{1+r} - L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L' - (1+r)\frac{(1+r_\ell)(1-\varphi)L'}{1+r}\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\varphi'}{(1+r)-(1+r_\ell)(1-\varphi')}\right) + \frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

$$\begin{aligned}
\Psi(L,B) &= \max_{L'} \log\left((1+r_\ell)L - (1+r)B - \frac{(1+r)-(1+r_\ell)(1-\varphi)}{1+r}L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)\varphi L'\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\varphi'}{(1+r)-(1+r_\ell)(1-\varphi')}\right) + \frac{\beta_L\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}$$

The first-order condition is

$$\frac{\frac{(1+r)-(1+r_\ell)(1-\phi)}{1+r}}{(1+r_\ell)L-(1+r)B-\frac{(1+r)-(1+r_\ell)(1-\phi)}{1+r}L'}=\frac{\beta_L}{1-\beta_L}\frac{1}{L'},$$

which gives

$$\begin{aligned} L' &= \frac{\beta_L(1+r)}{(1+r)-(1-\phi)(1+r_\ell)}\left((1+r_\ell)L-(1+r)B\right) \\ B' &= \frac{\beta_L(1-\phi')(1+r_\ell)}{(1+r)-(1-\phi')(1+r_\ell)}\left((1+r_\ell)L-(1+r)B\right). \end{aligned}$$

Given these decision rules, the value function is given by

$$\begin{aligned} \Psi(L,B) &= \frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right) \\ &+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi')}\right) \\ &+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right)+\frac{\log(1-\beta_L)}{1-\beta_L}. \end{aligned}$$

Equating this expression to our initial guess,

$$\frac{1}{1-\beta_L}\log\left((1+r_\ell)L-(1+r)B\right)+\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi)}\right)+\frac{\log(1-\beta_L)}{1-\beta_L},$$

we obtain

$$\begin{aligned} \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi)}\right) &= \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi}{(1+r)-(1+r_\ell)(1-\phi)}\right) \\ &+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r_\ell)(1+r)\beta_L\phi'}{(1+r)-(1+r_\ell)(1-\phi')}\right), \end{aligned}$$

which gives

$$\frac{\phi}{(1+r)-(1+r_\ell)(1-\phi)}=\frac{\phi'}{(1+r)-(1+r_\ell)(1-\phi')}.$$

Since these expressions are monotone (and declining) in ϕ, they imply that $\phi=\phi'$.

By imposing this into

$$\phi = \xi \left(\frac{1 + r - (1 + r_\ell)(1 - \phi')}{(1 + r)\phi'} \right)^{\frac{\beta_L}{1-\beta_L}}.$$

we obtain

$$\phi = \xi^{1-\beta_L} \left(\frac{1 + r - (1 + r_\ell)(1 - \phi)}{(1 + r)} \right)^{\beta_L}.$$

□

I.2 Transition

Assume that the last period of the transition is period T and the economy is in steady state with r_ℓ and r from period $T + 1$ and onward. The following proposition characterizes the bank's solution in the transition, where all prices $r_{\ell,t}$ and r_t are potentially changing.

Proposition 2. *The solution to the bank's problem is given as follows:*

1. The value function:

$$\Psi_t (L_t, B_t) = \frac{1}{1 - \beta_L} \log \big((1 + r_{\ell,t})L_t - (1 + r_t)B_t \big) + \Omega_t + \frac{\log(1 - \beta_L)}{1 - \beta_L},$$

   where

$$\Omega_t = \frac{\beta_L}{1 - \beta_L} \log \left(\frac{\beta_L \phi_{t+1}(1 + r_{t+1})1 + r_{\ell,t+1}}{1 + r_{t+1} - (1 - \phi_{t+1})1 + r_{\ell,t+1}} \right) + \beta_L \Omega_{t+1};$$

$$\Omega_T = \Omega = \frac{\beta_L}{(1 - \beta_L)^2} \log \left(\frac{\beta_L \phi(1 + r)\,(1 + r_\ell)}{1 + r - (1 - \phi)\,(1 + r_\ell)} \right);$$

$$\phi_t = \xi^{1-\beta_L} \left(\frac{1 + r_{t+1}}{1 + r_{\ell,t+1}} - (1 - \phi_{t+1}) \right)^{\beta_L};$$

   and

$$\phi_T = \phi.$$

2. The no-default constraint in period t can be written as

$$\big(1 + r_{\ell,t+1}\big)\,(1 - \phi_{t+1})\,L_{t+1} \geqslant (1 + r_{t+1})\,B_{t+1}.$$

3. The bank's solution satisfies the following expression regardless of whether or not the no-default constraint binds:

$$L_{t+1} - B_{t+1} = \beta_L \big((1 + r_{\ell,t})L_t - (1 + r_t)B_t \big).$$

4. The decision rules when the no-default constraint is binding (if $r_{\ell,t+1} > r_{t+1}$):

$$\begin{aligned} L_{t+1} &= \frac{\beta_L(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right) \\ B_{t+1} &= \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right). \end{aligned}$$

5. The decision rules when the no-default constraint is not binding (if $r_{\ell,t+1} \leqslant r_{t+1}$):

$$B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right)\right] & \text{if } r_{\ell,t+1} = r_{t+1} \\ 0 & \text{if } r_{\ell,t+1} < r_{t+1} \end{cases}$$

and

$$L_{t+1} = B_{t+1} + \beta_L\left((1+r_{\ell,t})L_t-(1+r_t)B_t\right).$$

Proof. We are going to solve the problem backward starting from period T.

Period T:

$$\begin{aligned} \Psi_T(L_T, B_T) &= \max_{L_{T+1}, B_{T+1}} \log\left((1+r_{\ell,T})L_T-(1+r_T)B_T-(L_{T+1}-B_{T+1})\right) \\ &+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_\ell)L_{T+1}-(1+r)B_{T+1}\right) \\ &+ \left(\frac{\beta_L}{1-\beta_L}\right)^2 \log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right) + \frac{\beta_L}{1-\beta_L}\log(1-\beta_L) \\ &\text{s.t.} \\ (1-\phi)(1+r_\ell)L_{T+1} &\geqslant (1+r)B_{T+1}. \end{aligned}$$

The decision rules of this problem are given as

$$\begin{aligned} L_{T+1} &= \frac{\beta_L(1+r)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\ B_{T+1} &= \frac{\beta_L(1-\phi)(1+r_\ell)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\ L_{T+1}-B_{T+1} &= \beta_L\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \\ (1+r_\ell)L_{T+1}-(1+r)B_{T+1} &= \frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\left((1+r_{\ell,T})L_T-(1+r_T)B_T\right) \end{aligned}$$

which give

$$\begin{aligned}\Psi_T\left(L_T, B_T\right) &= \frac{1}{1-\beta_L}\log\big((1+r_{\ell,T})L_T-(1+r_T)B_T\big)\\ &+ \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).\end{aligned}$$

The value function when the bank defaults is

$$\tilde{\Psi}_T^D\left(\xi(1+r_{\ell,T})L_T\right)=\frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T})L_T\right)+\frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r))+\frac{\log(1-\beta_L)}{1-\beta_L}.$$

The no-default condition in period T can be written as

$$(1-\phi_T)(1+r_{\ell,T})L_T \geqslant (1+r_T)B_T,$$

where

$$\phi_T=\xi^{1-\beta_L}\left(\frac{1+r}{1+r_\ell}-(1-\phi)\right)^{\beta_L}.$$

Period $T-1$:

$$\begin{aligned}\Psi_{T-1}\left(L_{T-1}, B_{T-1}\right) &= \max_{L_T,B_T}\log\big((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}-(L_T-B_T)\big)\\ &+ \frac{\beta_L}{1-\beta_L}\log\big((1+r_{\ell,T})L_T-(1+r_T)B_T\big)\\ &+ \left(\frac{\beta_L}{1-\beta_L}\right)^2\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{\beta_L}{1-\beta_L}\log(1-\beta_L)\\ &\text{s.t.}\\ (1-\phi_T)(1+r_{\ell,T})L_T &\geqslant (1+r_T)B_T.\end{aligned}$$

The decision rules for this problem are given as

$$\begin{aligned}L_T &= \frac{\beta_L(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\big((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\big)\\ B_T &= \frac{\beta_L(1-\phi_T)(1+r_{\ell,t})}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\big((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\big)\\ L_T-B_T &= \beta_L\big((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\big)\\ (1+r_{\ell,t})L_T-(1+r_T)B_T &= \frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\big((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\big),\end{aligned}$$

which give

$$\begin{aligned}\Psi_{T-1}\left(L_{T-1}, B_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\ &+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right)\\ &+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).\end{aligned}$$

The value function when the bank defaults is

$$\begin{aligned}\tilde{\Psi}^D_{T-1}\left(\xi(1+r_{\ell,T-1})L_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T-1})L_{T-1}\right)+\frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r_T^D))\\ &+ \frac{\beta_L^2}{(1-\beta_L)^2}\log(\beta_L(1+r^D))+\frac{\log(1-\beta_L)}{1-\beta_L}.\end{aligned}$$

The no-default condition in period $T-1$ can be written as

$$(1-\phi_{T-1})(1+r_{\ell,T-1})L_{T-1}\geqslant(1+r_{T-1})B_{T-1},$$

where

$$\phi_{T-1}=\xi^{1-\beta_L}\left(\frac{1+r_T}{1+r_{\ell,t}}-(1-\phi_T)\right)^{\beta_L}.$$

Period $T-2$:

$$\begin{aligned}\Psi_{T-2}\left(L_{T-2}, B_{T-2}\right) &= \max_{L_{T-1},B_{T-1}}\log\left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}-(L_{T-1}-B_{T-1})\right)\\ &+ \frac{\beta_L}{1-\beta_L}\log\left((1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\ &+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right)\\ &+ \frac{\beta_L^3}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{\beta_L}{1-\beta_L}\log(1-\beta_L)\\ &\text{s.t.}\\ (1-\phi_{T-1})(1+r_{\ell,T-1})L_{T-1} &\geqslant (1+r_{T-1})B_{T-1}.\end{aligned}$$

The decision rules of this problem are given as

$$\begin{aligned}
L_{T-1} &= \frac{\beta_L(1+r_{T-1})}{1+r_{T-1}-(1-\phi)(1+r_{\ell,T-1})}\omega_{t-2} \\
B_{T-1} &= \frac{\beta_L(1-\phi_{T-1})(1+r_{\ell,T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\omega_{t-2} \\
L_{T-1}-B_{T-1} &= \beta_L\omega_{t-2} \\
(1+r_{\ell,T-1})L_{T-1}-(1+r_{T-1})B_{T-1} &= \frac{\beta_L\phi_{T-1}(1+r_{\ell,T-1})(1+r_{T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\omega_{t-2}, \\
\omega_{t-2} &= \left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}\right),
\end{aligned}$$

which give

$$\begin{aligned}
\Psi_{T-2}(L_{T-2},B_{T-2}) &= \frac{1}{1-\beta_L}\log\left((1+r_{\ell,T-2})L_{T-2}-(1+r_{T-2})B_{T-2}\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_{T-1}(1+r_{\ell,T-1})(1+r_{T-1})}{1+r_{T-1}-(1-\phi_{T-1})(1+r_{\ell,T-1})}\right) \\
&+ \frac{\beta_L^2}{1-\beta_L}\log\left(\frac{\beta_L\phi_T(1+r_{\ell,t})(1+r_T)}{1+r_T-(1-\phi_T)(1+r_{\ell,t})}\right) \\
&+ \frac{\beta_L^3}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r_\ell)(1+r)}{1+r-(1-\phi)(1+r_\ell)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).
\end{aligned}$$

The value function when the bank defaults is

$$\begin{aligned}
\tilde{\Psi}^D_{T-2}\left(\xi(1+r_{\ell,T-2})L_{T-2}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r_{\ell,T-2})L_{T-2}\right)+\frac{\log(1-\beta_L)}{1-\beta_L} \\
&+ \frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r_{T-1}))+\frac{\beta_L^2}{1-\beta_L}\log(\beta_L(1+r_T)) \\
&+ \frac{\beta_L^3}{(1-\beta_L)^2}\log(\beta_L(1+r)).
\end{aligned}$$

The no-default condition in period $T-2$ can be written as

$$(1-\phi_{T-2})(1+r_{\ell,T-2})L_{T-2}\geqslant(1+r_{T-2})B_{T-2},$$

where

$$\phi_{T-2}=\xi^{1-\beta_L}\left(\frac{1+r_{T-1}}{1+r_{\ell,T-1}}-(1-\phi_{T-1})\right)^{\beta_L}.$$

The derivations suggest that the value functions and decision rules have the same pattern. Thus, they will take the same form of the previous period. □

I.3 Bank's solution

Given the collateral constraint the bank is facing, we can explicitly solve for the bank's problem, which is summarized in the following proposition.

Proposition 3. *The decision rules when the no-default constraint binds (if* $r_{\ell,t+1} > r_{t+1}$*) are*

$$
\begin{aligned}
L_{t+1} &= \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\beta_L\omega_t \\
B_{t+1} &= \frac{(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\beta_L\omega_t,
\end{aligned}
$$

where $\omega_t = (1+r_{\ell,t})L_t - (1+r_t)B_t$.

The decision rules when the no-default constraint does not bind (if $r_{\ell,t+1} \leqslant r_{t+1}$*) are:*

$$
B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r_{\ell,t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}\omega_t\right] & \text{if } r_{\ell,t+1} = r_{t+1} \\ 0 & \text{if } r_{\ell,t+1} < r_{t+1} \end{cases}
$$

and

$$
L_{t+1} = B_{t+1} + \beta_L\left((1+r_t^*)L_t - (1+r_t)B_t\right).
$$

I.4 Characterization of the Bank's Problem in Stationary Equilibrium

We can further characterize the bank's problem under stationarity. Throughout the paper, we will focus on stationary equilibria where the capital requirement constraint is binding. If it did not bind, then bank balance sheets would not have any impact on the economy. However, we do not rule out the case that there might be some periods in the transition where this constraint becomes slack. Using the general formula capturing both the exogenous and endogenous capital requirement constraint, we have the following decision rules when the constraint binds:

$$
L_{t+1} = \beta_L\widehat{\lambda}_t\omega_t \quad \text{and} \quad B_{t+1} = \beta_L\left(\widehat{\lambda}_t - 1\right)\omega_t,
$$

where

$$
\widehat{\lambda}_t = \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})}. \tag{I.1}
$$

Then the law of motion for net worth is given as

$$
\omega_{t+1} = L_{t+1}\left(1+r_{\ell,t+1}\right) - B_{t+1}\left(1+r_{t+1}\right).
$$

Then, we can obtain the next period's net worth as

$$\omega_{t+1} = \beta_L \left(\widehat{\lambda}_t \left(1 + r_{\ell,t+1}\right) - \left(\widehat{\lambda}_t - 1\right) \left(1 + r_{t+1}\right) \right) \omega_t.$$

Imposing steady state $\omega_{t+1} = \omega_t$ and $\widehat{\lambda}_t = \widehat{\lambda}$ gives

$$r_\ell - r = \frac{1 - \beta_L(1 + r)}{\widehat{\lambda}\beta_L},$$

where $r_\ell - r$ is the premium due to the bank capital constraint. If $\beta_L(1 + r) < 1$ and $\widehat{\lambda} < \infty$, then $r_\ell - r > 0$. Thus, the capital constraint will be binding in the stationary equilibrium. To understand this point, assume that $\beta_L(1 + r) < 1$ but the bank starts with a high net worth so that the capital requirement constraint is not binding. In that case, $r_{\ell,t+1} = r$ and the bank's decision rule is $L_{t+1} - B_{t+1} = \beta_L \omega_t$. Using that, we can show that $\omega_{t+1} = (1 + r) \beta_L \omega_t < \omega_t$. Thus, the bank econsumes from its net worth until the capital constraint starts to bind. Thus, the economy will converge to a stationary equilibrium where it actually binds.

J Computational Algorithm

Denote the state variable of the household as $\theta = (a, h, d, z, j, s)$ where s is the housing tenure, j is the age of the household, z is the income efficiency shock, d is the ratio of mortgage debt to initial house price level, h is the size of the owner-occupied unit, and a is the financial wealth after the return is realized. For active/inactive renters ($s \in \{r, i\}$) $h = d = 0$. We discretize a into 120 and d into 60 exponentially spaced points. The age j runs from 1 to 30, and h is linearly discretized into 5 points. Income shock z is discretized into 15 points, and grid points and transition probabilities are computed using the Tauchen method. Since this is a life-cycle model, the grid points for income shocks are age dependent to better approximate the AR(1) process with a Markov process. We approximate the US retirement system following Guvenen and Smith (2014). We adjust the retirement income level such that working age-households pay 12 percent tax.

J.1 Steady-State Computation

The steady state of the model is computed as follows:

1. From the bank's problem, the lending rate at the steady state is $r_\ell = r + \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$.

2. Make a guess on K and p_h.

3. Given these guesses, using the firm's problem, compute w and u:

$$
\begin{aligned}
u &= \left(\frac{(1-\alpha)K}{(1+\phi r_\ell)\vartheta}\right)^{\frac{1}{\alpha+\psi}} \\
w &= \vartheta^{\frac{\alpha-1}{\alpha+\psi}}\left(\frac{(1-\alpha)K^\alpha}{(1+\phi r_\ell)}\right)^{\frac{1+\psi}{\alpha+\psi}} \\
r_k &= \alpha\left(\frac{K}{u}\right)^{\alpha-1} - \delta
\end{aligned}
$$

4. Using the rental companies' problem, compute the rent price:

$$p_r = \kappa + \frac{1-\delta_h}{1+r_k}p_h$$

5. Given all these prices, solve the household's problem recursively:

   (a) Solve the terminal period problem where all dynamic choices are set to 0: $a' = d' = 0$. This gives the value for the household, $V_J(\theta)$, and the continuation value of the mortgage contract, $v_J^l(\theta)$.

   (b) Given $V_j(\theta)$ and $v_j^l(\theta)$, solve $V_{j-1}(\theta)$ and $v_{j-1}^l(\theta)$:

      i. Given $V_j(\theta)$ and $v_j^l(\theta)$, first solve the expected continuation values $EV_j(\theta)$ and $Ev_j^l(\theta)$.

      ii. Solve for mortgage prices at the origination, $q^m(\theta)$.

      iii. The solutions to the problems for the inactive renter and the active renter who decides to become a renter are straightforward. Their choices are housing services, consumption, and saving. We interpolate the expected value of the continuation value using linear interpolation, and to choose the optimal saving level, we first search globally over a finer discrete space for a' to bracket the maximum.[41] Once the maximum is bracketed, we solve for the optimum using Brent's method. Given the saving choice, we compute the optimal housing services using the analytical expression for it.[42] Then, we use the budget constraint to compute the consumption.

      iv. The most complex and time-consuming problem is the problem of the renter who decides to purchase a house. This household chooses consumption, saving, house size, and mortgage debt. We restrict the choice of down payment and house size to finite sets. For down

[41] For saving choice, we use 240 grid points.

[42] Since the utility function is CES in consumption and housing services, we can obtain an analytical expression for optimal housing services.

payment, the grid points for d are the choices, and for house size the grid points for h are the choices.[43] For each down payment and house size choices, we solve the household's objective function, $V_{j-1}^{d,h}$, by finding the optimal saving level, as we discussed in point 5(b)iii. Given all household choices, we can obtain $q^m(\theta)$. We use linear interpolation for the points off the grid. Also given the choice of d and h, the mortgage debt becomes $dp_h^* h$ where p_h^* is the equilibrium price level at the initial steady state. Once the objective function is solved for a given down payment and house size choice, we set $V_{j-1}(\theta) = max_{d,h}\left\{V_{j-1}^{d,h}\right\}$.

v. The solution of the homeowner's problem:

   A. Stayer: The stayer's problem is simple since the household only chooses consumption and saving. We solve it similar to the inactive renter's problem. The only exception is that in the continuation value, the variable keeping track of the principal amount d will be adjusted. Given current d, $d' = (d - m)(1 + r_\ell)$ where $m = \frac{r_\ell(1+r_\ell)^{J-j}}{(1+r_\ell)^{J-j+1}-1}$. We use linear interpolation over d' to compute the expected continuation value for the household.

   B. Seller: The seller's problem is the same as the problem of an active renter except that in the budget constraint, the household will have the term due to the proceedings from the sale of the house: $p_h h(1-\varphi_s) - dhp_h^*$

   C. Refinancer: The refinancer's problem is the same as the problem of a renter who purchases a house except that she is restricted to purchasing the same house.

   D. Defaulter: The defaulter's problem is the same as the active renter's problem.

vi. Solving the homeowner's problem also gives us the mortgage payment for each type of mortgage contract and allows us to compute the continuation of the mortgage contract, $v_j^l(\theta)$:

$$v_{j-1}^l(\theta) = m(\theta) + \frac{1}{1+r_\ell}\int_{\theta'} v_j^l(\theta')\,\Pi(\theta'|\theta),$$

where

$$m(\theta) = \begin{cases} dhp_h^* & \text{if } s \in \{hr, hf\} \\ p_h h(1-\varphi_e) & \text{if } s = he \\ \frac{r_\ell(1+r_\ell)^{J-j}}{(1+r_\ell)^{J-j+1}-1} dhp_h^* & \text{if } s = hh \end{cases}$$

[43]Increasing the number of grid points for d and h beyond the levels we set does not noticeably change the results.

(c) Repeat step (b) for each $j = \{J-1, ..., 1\}$.

6. Given the policy functions for the household, simulate the economy $N = 20,000$ individuals for $J = 30$ periods. This gives us aggregate saving, A, aggregate housing demand, H^d, and aggregate rental demand, H^r. Given aggregate saving, we update the aggregate capital guess as $K = (1-\lambda_k) K + \lambda_k (A - V^{rc}(H^r))$ where $V^{rc} = \frac{p_r - \kappa - \delta_h p_h^0}{r_k} H^r$ is the value of rental companies. Given aggregate housing demand, we update the house price guess as $p_h = p_h \left(1 + \lambda_h \frac{H - \bar{H}}{\bar{H}}\right)$. We set $\lambda_k = \lambda_h = 0.1$. We continue this process until $max\left(|A - W(H^r) - K|, \left|H - \bar{H}\right|\right) < \epsilon$ where $\epsilon = 10^{-4}$.

7. Once equilibrium prices and allocations are solved, we solve for bank-related variables: bank net worth, bank assets, and bank liabilities using the steady-state analytical equations for these variables.

J.2 Transition Algorithm

The transitional problem has two main differences. First, we need to solve for a path of equilibrium prices and allocations along the transition. Second, we need to adjust the algorithm to capture the fact that the risk-free mortgage interest rate can change along the transition. This second point is important because in order to save from an additional state variable, we assume individuals pay points at the origination time to reduce the risk-adjusted mortgage interest rate to the risk-free mortgage interest rate. This allows us to eliminate the mortgage interest rate as an additional state variable. However, since shocks are permanent, this assumption can artificially distort the equilibrium. Consider a decline in the risk-free mortgage interest rate from 5 percent to 4 percent. If we still assume all new mortgages are priced at 5 percent, this would imply that banks would be paid more than the principal amount if they still use the same amortization schedule we use in the steady-state algorithm. That will result in q^m being significantly larger than 1, implying a substantial subsidy from banks to individuals. More importantly, if we also apply this new risk-free mortgage interest rate to existing mortgages, that would imply a reduction of all the existing mortgage payments: a positive wealth shock to all existing mortgage owners and a negative shock to banks.[44]

To tackle this issue without further complicating the solution algorithm, we assume that after the shock is realized, all new mortgages will be priced at the new risk-free mortgage rate, whereas all existing mortgages will still be paid using

[44] Since we keep track of the principal balance as a state variable, we need to know the risk-free mortgage rate to compute the implied mortgage payments. Another formulation could be to keep track of the mortgage payments. However, in this case, we still need to know the risk-free mortgage rate in order to compute the implied principal amount since it affects the resources of homeowners in the event of selling/refinancing/defaulting.

the old risk-free mortgage rate. We also include an additional state variable in the household's problem to keep track of whether the household purchased a house before or after the shock is realized. This allows us to compute the mortgage payments more accurately without substantially distorting the solution algorithm.

Given these modifications, the rest of the algorithm is as follows:

1. Fix the time it takes for the transition to happen: T periods. We set $T = 60$ corresponding to 120 years.

2. Solve the initial steady state of the problem as outlined above. Store the initial steady-state distribution denoted as $\Gamma_0(\theta)$.

3. Given the boom shock, solve the final steady state of the problem as outlined above. Store $V_T(\theta)$ and $\nu_T^l(\theta)$.

4. Guess the path of aggregate capital stock, rental demand, house price, and lending rate: $\left\{K_{t+1}, H_t^{r,0}, p_t^h, r_{\ell,t+1}^0\right\}_{t=1}^{T-1}$.

5. Given these guesses, compute $\left\{w_t, r_{k,t+1}, p_t^r\right\}$ using the good-producing firm's and rental companies' problem. Compute V_t^{rc} using the rental companies' problem.

6. Solve each cohort's problem for each period they are alive, starting from the cohort born in period $-J+2$ until the cohort born in period $T-1$[45]:

   (a) For each generation, given prices, solve the household's problem and the continuation value of the contract as in the steady-state problem above. The only difference is that for new mortgage buyers, the risk-free mortgage interest rate is the final steady-state risk-free mortgage interest rate, whereas for existing mortgage owners, it is the initial steady-state risk-free mortgage interest rate. This also affects the continuation value for households and mortgage contracts since we need to keep track of whether a mortgage originated before or after the shock.

   (b) Given the policy functions for each generation, simulate the economy starting from the initial steady-state distribution $\Gamma_0(\theta)$ for T periods. We fix the same random numbers for the idiosyncratic shocks to household.

   (c) Using the simulated path, compute the aggregates: $A_{t+1}, H_t^{r,1}, H_t^d$, $M_t = \int \nu_t^l(\theta)$.

[45] A household of age j belonging to a cohort born in period $g \in \{-J+2, ..., T-1\}$ will be subject to prices p_{g+j-1}.

(d) Update guesses:

$$\begin{aligned}
K_{t+1} &= (1-\lambda_k) K_{t+1} + \lambda_k \left(A_{t+1} - V^{rc}_{t+1}\left(H^r_{t+1}\right)\right) \\
H^r_t &= (1-\lambda_{rc}) H^{r,0}_t + \lambda_{rc} H^{r,1}_t \\
p^h_t &= p^h_t \left(1 + \lambda_h \frac{H^d_t - \bar{H}}{\bar{H}}\right) \\
r^0_{\ell,t+1} &= (1-\lambda_r) r^0_{\ell,t+1} + \lambda_{rc} r^1_{\ell,t+1}
\end{aligned}$$

where $r_{\ell,t+1}$ solves

$$L_{t+1} = \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{\ell,t+1})} \beta_L \omega_t$$

where $L_{t+1} = M_{t+1} + \phi w_{t+1}(\bar{w}, u_{t+1})$ and

$$\omega_t = \begin{cases} L_t\left(1+r_{\ell,t}\right) - B_t\left(1+r_t\right) & \text{if } t=1 \\ \left(1+r^0_{\ell,t}\right)\phi_t L_t & \text{if } t>1. \end{cases}$$

(e) Iterate this process until convergence occurs on guesses. The convergence criteria are defined as $\max\left|K_{t+1} + V^{rc}_{t+1}\left(H^r_{t+1}\right) - A_{t+1}\right| < \epsilon_k$, $\max\left|H^{r,1}_t - H^{r,0}_t\right| < \epsilon_h$, $\max\left|H^d_t - \bar{H}\right| < \epsilon_h$, and $\max\left|r^1_{\ell,t} - r^0_{\ell,t}\right| < \epsilon_r$ where $\epsilon_k = \epsilon_h = 10^{-3}$ and $\epsilon_r = 10^{-4}$.