Joint-Search Theory:
New Opportunities and New Frictions

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Abstract

Search theory routinely assumes that decisions about the acceptance/rejection of job offers (and, hence, about labor market flows between jobs or across employment states) are made by individuals acting in isolation. In reality, the vast majority of workers are somewhat tied to their partners—in couples or families—and decisions are made jointly. This paper studies, from a theoretical viewpoint, the joint job-search and location problem of a household formed by a couple (e.g., husband and wife) who perfectly pools income. The objective, in the spirit of standard search theory, is to characterize the reservation wage behavior of the couple and compare it to the single-agent search model in order to understand the ramifications of partnerships for individual labor market outcomes and wage dynamics. We focus on two main cases. First, when couples are risk averse and pool income, joint search yields new opportunities—similar to on-the-job search—relative to single-agent search. Second, when the two spouses in a couple face job offers from multiple locations and a cost of living apart, joint search features new frictions and can lead to significantly worse outcomes than single-agent search.

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1 Introduction

Macroeconomics is rapidly shifting away from the stylized "bachelor model" of the household to models that explicitly recognize the relevance of within-household decisions for aggregate economic outcomes.[1] Surprisingly, instead, search theory has almost entirely focused on the single-agent search problem, since its inception in the early 1970. The recent comprehensive survey by Rogerson, Shimer, and Wright (2005), for example, does not contain any discussion on optimal job search strategies of two-person households acting as the decision units. This state of affairs is rather surprising given that Burdett and Mortensen (1977), in their seminal piece entitled "Labor Supply Under Uncertainty," sketch a characterization of a two-person search problem, explicitly encouraging further work on the topic. Their pioneering effort, which remained virtually unfollowed, represents the starting point of our theoretical analysis.

In this paper, we study the job search problem of a couple who faces exactly the same economic environment as in the standard single-agent search problem of McCall (1970) and Mortensen (1970) without on-the-job search, and of Burdett (1978) with on-the-job search. A couple is an economic unit composed of two ex-ante identical individuals linked by the assumption of perfect income pooling. The simple unitary model of a household adopted here is a convenient and logical starting point. It helps us to examine transparently the role of labor market frictions and insurance opportunities introduced by joint-search, and it makes the comparison with the canonical single-agent search model especially stark.

From a theoretical perspective, couples may make joint decisions (leading to choices different from those of a single agent) for several reasons. We start from the two most natural and relevant ones. First, the couple has concave utility over pooled income. Second, the couple can receive job offers from multiple locations, but faces a cost of living apart. In the latter case, deviations from the single-agent search problem occur even for linear preferences. As summarized by the title of our paper, in the first environment joint search introduces new

[1]For example, see Aiyagari et al. (2000) on intergenerational mobility and investment in children, Cubeddu and Rios-Rull (2003) on precautionary saving, Blundell et al. (2007) on labor supply, Heathcote et al. (2010) and Lise and Seitz (2011) on economic inequality, and Guner et al. (2010) on taxation.

opportunities, whereas in the second it introduces new frictions relative to single-agent search. The set of propositions we prove characterizes optimal behavior in terms of comparison between the reservation wage functions of the couple and the reservation wage value of the single agent. One appealing feature of our theoretical analysis is that it yields two-dimensional diagrams in the space of the two spouses' wages (w_1, w_2), where the reservation wage policies can be easily analyzed and interpreted.

In the first environment we study, couples are risk-averse and the economy has one location only. A dual-searcher couple (both members unemployed) will quickly accept a job offer—in fact, faster than a single unemployed agent. The dual-searcher couple can use income pooling and joint search to its advantage: it initially accepts a lower wage offer (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income) which remains available to the other spouse. Once a worker-searcher couple (one spouse employed, the other unemployed), the pair will be more choosy in accepting the subsequent job offers. We formally show that the shape of the reservation wage of the worker-searcher couple (a function of the employed spouse's wage) depends on how absolute risk aversion changes with the level of consumption.

A key feature of the solution to the joint-search problem is that the searching spouse accepting a job offer may trigger a quit by the employed spouse motivated by the search for a better job. The outcome of this behavior is a switch between the breadwinner and the searcher within the household. As is well known, endogenous quits never happen in the corresponding single-agent version of the search model. We call this process—of work-quit-search-work that allows a couple to climb the wage ladder even in absence of on-the-job search—the "breadwinner cycle." Therefore, one can view joint search as a "costly" version of on-the-job search, even in its formal absence. The cost comes from the fact that, in order to keep the search option active, the pair must remain a worker-searcher couple, and cannot enjoy the full wage earnings of a dual-worker couple as it would be capable of doing in the presence of on-the-job search.

Overall, relative to singles, couples spend more time searching for better jobs, which results in longer unemployment durations, but eventually leads to higher lifetime wages and welfare (whence, the "new opportunities" in the title of the paper). Quantitatively, deviations of joint-search behavior from its single-agent counterpart can be substantial. For example, a plausible calibration of the model implies that each spouse in a couple earns a lifetime income that is 1-2 percent higher than a comparable single agent. Using micro data from the Survey of Income and Program Participation (SIPP), which tracks weekly employment histories of all household members, we show that some key empirical stylized facts about joint search (e.g., frequency of breadwinner cycles, and mean unemployment durations of different household types) are quantitatively in line with simulations of the model with CRRA utility and risk aversion coefficient around two.

Our second environment features multiple locations and a flow cost of living apart for each of the spouses in the couple. The couple has to choose reservation wages with respect to "inside offers" (jobs in the current location) and "outside offers" (jobs in other locations). Even with risk-neutral preferences, the search behavior of couples differs from that of single agents in important ways. First, the dual-searcher couple is less choosy than the individual agent because it is effectively facing a worse job offer distribution, since some wage offer configurations are attainable only in different locations—hence, by paying the cost of living apart. Second, there is a region in which the breadwinner cycle is optimal for the couple. For example, a couple who gets a very generous job offer from the outside location could be better off if the currently employed spouse quits and follows the spouse with the job offer to the new location. It should be noted that we also obtain these two results—couples being less picky than singles and the breadwinner cycle—in our previous environment, but for completely different reasons.

The model allows us to formalize what Mincer (1978) called tied-stayers—i.e., workers who turn down a job offer from a different location that they would accept if single—and tied-movers—i.e., workers who accept a job offer in the location of the partner that they would turn down if single. Overall, the disutility of living separately shrinks the set of job offers that are viable for couples, compared to that of singles (whence, the "new frictions" in the title of the paper).

The relevance of a multiple-location joint-search model of the labor market is supported, for example, by Costa and Kahn (2000) who document that highly educated dual-career couples have increasingly relocated to large metropolitan areas in the United States since the 1960s (more so than comparable singles); cities offer a greater and more diverse set of job opportunities, thereby mitigating the frictions associated with joint search.

Also for the multiple-location model, deviations of joint-search behavior from its single-agent counterpart can be quantitatively substantial. For example, when the (flow) disutility cost of living separately is equal to 15% of a couple's earnings, half of all households moving across locations comprise a partner who is a tied-mover, and the lifetime income of each spouse in a couple is 6.6% lower than comparable singles.

We conclude this Introduction by briefly reviewing the related literature. Only very recently, a handful of papers have started to follow the lead offered by Burdett and Mortensen (1977) into the investigation of household interactions in frictional labor market models. Garcia-Perez and Rendon (2004) numerically simulate a model of family-based job-search decisions to tease out the importance of the added worker effect for consumption smoothing. Dey and Flinn (2008) study quantitatively the effects of health insurance coverage on employment dynamics in a search model where the economic unit is the household. Gemici (2011) estimates a rich structural model of migration and labor market decisions of couples to assess the implications of joint location constraints on labor outcomes and the marital stability of couples. Flabbi and Mabli (2011) focus on the bias in estimates of structural search parameters when the model is misspecified because it ignores the joint-search component. Relative to these contributions, our paper is less ambitious in its quantitative analysis, but provides a more focused and systematic study of joint-search theory.

From a theoretical perspective, our analysis of the one-location model has useful points of contact with existing results in search theory applied to at least two separate contexts. First, starting from the static analysis of Danforth (1979), a number of papers have studied the role of risk-free wealth in shaping dynamic job-search decisions (e.g., Andolfatto and Gomme, 1996; Gomes, Greenwood, and Rebelo, 2001; Pissarides, 2004; Lentz and Tranaes, 2005; Browning et al., 2007). The income of the spouse differs crucially from risk-free wealth because it is risky (in the presence of exogenous separations) and because it can be optimally controlled by the job-search decision itself. Second, Albrecht, Anderson, and Vroman (2010) study a different type of joint-search decision, that of a committee voting on an option which gives some value to each member. The authors are interested in drawing a comparison between single-agent search and committee search, in the same spirit as our exercise.[2]

The rest of the paper is organized as follows. Section 2 lays out the single-agent problem which provides the benchmark of comparison throughout the paper. Section 3 analyzes the baseline joint-search problem as well as some extensions. Section 4 shows that simulations from a calibrated model yields implications broadly in line with stylized facts about joint-search documented from SIPP data. Section 5 studies the joint-search problem with multiple locations. Section 6 concludes.

2 The Single-Agent Search Problem

We begin by presenting the sequential job-search problem of a single agent—the well-known McCall-Mortensen model (McCall, 1970; Mortensen, 1970). This model provides a useful benchmark against which we compare the joint-search model that we introduce in the next section. For clarity, we begin with a very stylized version and consider several extensions later.

Economic environment. Consider an economy populated by single individuals who all participate in the labor force: they are either employed or unemployed. Time is continuous and there is no aggregate uncertainty. Workers maximize the expected lifetime utility from consumption, $E_0 \int_0^\infty e^{-rt} u\left(c\left(t\right)\right) dt$, where r is the subjective rate of time preference, $c\left(t\right)$ is the

[2]The similarities, though, more or less stop here. For example, Albrecht, Anderson, and Vroman (2009) also find that committees are less picky than single agents. In our one-location model, this result is due to a consumption-smoothing argument. In their environment, it is due to the negative externality that committee members impose on each other.

consumption flow, and $u(\cdot)$ is the instantaneous utility function, which is strictly increasing, concave, and smooth.

An unemployed worker receives wage offers w at rate α from the exogenous distribution $F(w)$ with bounded support $[0, \bar{w}]$, and is entitled to a benefit flow $b \in (0, \overline{w})$. There is no recall of past wage offers. The worker observes the offer, w, and decides whether or not to accept it. If she rejects the offer, she continues to search. If she accepts the offer, she becomes employed at wage w forever, i.e., there are no exogenous separations and no new offers on the job. Individuals do not save or borrow.[3]

Value functions. Denote by V and W the value functions of an unemployed and employed agent, respectively. Then, using the continuous time Bellman equations, the search problem can be written in the following flow value representation:[4]

$$rV = u(b) + \alpha \int \max\{W(w) - V, 0\}\, dF(w) \tag{1}$$

$$rW(w) = u(w). \tag{2}$$

This well-known problem yields a unique reservation wage, w^*, such that for any wage offer above w^* the unemployed agent accepts the offer, and below w^*, she rejects the offer. This reservation wage is the solution to the equation

$$u(w^*) = u(b) + \frac{\alpha}{r} \int_{w^*} [u(w) - u(w^*)]\, dF(w). \tag{3}$$

The flow utility of accepting a job offer paying w^* (the left-hand side, LHS) is equated to the option value of continuing to search in the hope of obtaining a better offer (the right-hand

[3] As will become clear below, strictly speaking, in the present framework (without exogenous separation risk) we do not need to rule out saving: individuals face a wage earnings profile that is nondecreasing over the life cycle. As a result, since borrowing *is* ruled out, they would optimally set consumption equal to their wage earnings every period even though they were allowed to save. However, this is no longer true with exogenous separation, as we explain in Section 3.4.

[4] When the limits of integration are not specified, they are understood to be the lower and upper bound of the support of $F(w)$.

side, RHS). Since the LHS is increasing in w^* whereas the RHS is decreasing in w^*, and they are both continuous functions, equation (3) uniquely determines the reservation wage, w^*.

3 The Joint-Search Problem

We now study the search problem of a couple facing the same environment described above. A couple is a pair of ex-ante identical individuals who pool income to purchase a market good that is "public" within the household. As a result, there is no conflict between spouses in optimization.[5]

A couple can be in one of three labor market states. A "dual-searcher couple" is one where both spouses are unemployed and searching. A "dual-worker couple" is one where both spouses are employed (an absorbing state). A "worker-searcher couple" is one where one spouse is employed and the other is unemployed. As can perhaps be anticipated, the most interesting state is the last one.

Value functions. Let U denote the value function of a dual-searcher couple, $\Omega(w_1)$ the value function of a worker-searcher couple when the working spouse wage is w_1, and $T(w_1, w_2)$ the value function of a dual-worker couple earning wages w_1 and w_2. The value functions satisfy:

$$rT(w_1, w_2) = u(w_1 + w_2) \tag{4}$$

$$rU = u(2b) + 2\alpha \int \max\{\Omega(w) - U, 0\}\, dF(w) \tag{5}$$

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2). \tag{6}$$

The equations determining value functions (4) and (5) are straightforward analogs of their single-agent counterparts. When both spouses are employed, their flow value is simply determined by the total wage earnings of the household. When they are both unemployed, their flow value is equal to the instantaneous utility of consumption (which equals the total unem-

[5]Clearly, some consumption goods have marked features of public goods (e.g., housing services), others of private goods (e.g., food). In Section 3.5, we analyze the robustness of our results with respect to this assumption.

ployment benefit) *plus* the expected gain in case a wage offer is received. Because both agents receive new offers independently at rate α, the total offer arrival rate of a dual-searcher couple is 2α.[6]

The value function (6) of a worker-searcher couple is more involved. Upon receiving a wage offer the couple faces three choices. First, if the offer is rejected, there is no change in value. Second, if the offer is accepted and both spouses remain employed, the value increases by $T(w_1, w_2) - \Omega(w_1)$. Third, if the unemployed spouse accepts the wage offer w_2 *and* the employed spouse quits to search for a better one, the gain to the couple is $\Omega(w_2) - \Omega(w_1)$.

This last scenario is the crucial difference between the joint-search problem and the single-agent search problem. In the single-agent problem, once a job offer is accepted, the worker will never choose to quit. In contrast, in the joint-search problem, the reservation wage of each spouse depends on the income of the partner. When this income grows—for example, because of a transition from unemployment to employment—the reservation wage of the previously employed spouse may also increase, which could lead to exercising the quit option. Below, we return to this "endogenous nonstationarity" implicit in the joint-search problem.

3.1 Characterizing the Couple's Decisions

We are now ready to characterize the couple's search behavior. We begin with the following useful lemma. All proofs are contained in Appendix A.

Lemma 1 $\Omega(w)$ *is a continuous and strictly increasing function of* w.

Dual-Searcher Couple. For a dual-searcher couple, the reservation wage—which is the same for both spouses by symmetry—is denoted by w^{**} and is determined by the equation

$$\Omega(w^{**}) = U. \tag{7}$$

By virtue of Lemma 1, w^{**} is a singleton.

[6] In continuous time, the probability of both spouses receiving offers simultaneously is negligible.

Worker-Searcher Couple. As noted earlier, a worker-searcher couple's decision, upon receiving a wage offer w_2, involves choosing the highest among three values:

$$\max\left\{T\left(w_1, w_2\right), \Omega\left(w_2\right), \Omega\left(w_1\right)\right\},$$

which is a different way of writing the choices embedded in equation (6).

It is instructive to think of this problem in two stages. First, we ask: when does the couple *accept* a new wage offer? This happens if and only if w_2 is such that

$$\Omega\left(w_1\right) < T\left(w_1, w_2\right) \quad \text{or} \quad \Omega\left(w_1\right) < \Omega\left(w_2\right). \tag{8}$$

When this condition fails to hold, i.e., $\Omega\left(w_1\right) \geq \max\left\{T\left(w_1, w_2\right), \Omega(w_2)\right\}$, the couple will reject the offer. Second, *if* the offer is accepted—condition (8) *is* satisfied—the next question is, when does spouse 1 (currently employed) quit? A quit will happen if and only if

$$T\left(w_1, w_2\right) < \Omega\left(w_2\right). \tag{9}$$

For a given worker-searcher couple with current wage w_1, our goal is to find the threshold values that divide the range of w_2 into (potentially) three intervals: (i) one in which the offer is rejected ((8) fails to hold), (ii) another interval in which the offer is accepted and the employed spouse quits ((8) and (9) hold), and (iii) a third interval in which the offer is accepted but no quit takes place ((8) holds and (9) fails). We now characterize the reservation wage functions that determine these thresholds.

We start with the accept/reject decision described by condition (8). For every w_1, define $\phi^+\left(w_1\right)$ as the *lowest* wage offer that makes the couple weakly prefer $T\left(w_1, w_2\right)$ over $\Omega\left(w_1\right)$. Formally, this function solves

$$T\left(w_1, \phi^+\left(w_1\right)\right) = \Omega\left(w_1\right). \tag{10}$$

Similarly, define $\phi^{-}(w_1)$ to be the *lowest* wage offer that makes the couple weakly prefer $\Omega(w_2)$ over $\Omega(w_1)$. Then, $\phi^{-}(w_1)$ solves

$$\Omega\left(\phi^{-}(w_1)\right) = \Omega(w_1) \Rightarrow \phi^{-}(w_1) = w_1, \tag{11}$$

since Ω is invertible by Lemma 1. Thus, a wage offer w_2 that exceeds either one of the thresholds defined by (10) or (11) will be accepted. More formally, the reservation wage function for the accept/reject decision, $\phi(w_1)$, is defined as

$$\phi(w_1) \equiv \min\left\{\phi^{-}(w_1), \phi^{+}(w_1)\right\}. \tag{12}$$

We now turn to the stay/quit decision described by condition (9). A quit will never take place if the wage offer w_2 is rejected, as the couple would be worse off. Thus, consider a worker-searcher couple who has just received and accepted a wage offer w_2. Because the couple's income has changed with this decision, it will re-evaluate the wage of the employed spouse, w_1. As before, for every w_2, define the "quitting wage," $q(w_2)$, as the *highest* value of w_1 that makes the couple weakly prefer $\Omega(w_2)$ over $T(w_1, w_2)$. Formally, the associated indifference condition is

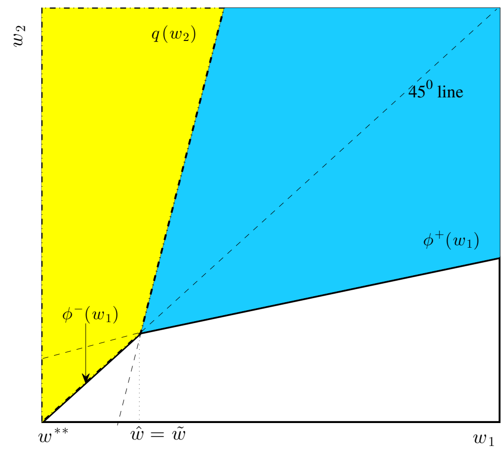
$$T(q(w_2), w_2) = \Omega(w_2). \tag{13}$$

Any value of $w_1 < q(w_2)$ satisfies condition (9) and triggers a quit. A comparison of (13) with (10) and the symmetry of the function T imply that $q(\cdot) \equiv \phi^{+}(\cdot)$—that is, the stay/quit decision is characterized by the same functional form as the accept/reject decision, except, of course, that the argument is w_1 in one case and w_2 in the other. This finding provides an important simplification in our analysis: we can focus on studying the properties of ϕ^{+}, knowing that the properties of q will simply follow from symmetry.

The following lemma is useful for the characterization of the reservation wage function ϕ.

Lemma 2 *There exists: (i) a wage $\hat{w} \geq w^{**}$ such that $\phi^{+}(w_1)$ and $\phi^{-}(w_1)$ intersect at $w_1 = \hat{w}$*

Figure 1: A Generic Diagram of Reservation Wage Functions for Worker-Searcher Couples



and, for all $w_1 < \hat{w}$, $\phi^+(w_1) > \phi^-(w_1)$, *and (ii) a wage* $\tilde{w} \in [\hat{w}, \overline{w})$ *such that, for all* $w_1 > \tilde{w}$, $\phi^+(w_1) < \phi^-(w_1)$ *and there are no quits.*

In light of (12), the main implication of this lemma is that, for $w_1 \leq \hat{w}$, the relevant reservation wage function is $\phi^-(w_1) = w_1$ (i.e., the 45^0–line in the (w_1, w_2) space), and for $w_1 > \tilde{w}$ the relevant reservation wage function is $\phi^+(w_1)$ and the quit option is never exercised—a useful result which simplifies many of our proofs below. [7]

Taking stock. It is helpful to visualize in the (w_1, w_2) space these various functions we have defined. Figure 1 shows a generic diagram of reservation wage functions for a worker-searcher couple. Throughout the paper, when we discuss worker-searcher couples, we will think of spouse 1 as the employed spouse and display his current wage w_1 on the horizontal axis, and think of spouse 2 as the unemployed spouse and display her offer, w_2, on the vertical axis.

The lowest possible wage at which one can observe a worker-searcher couple is w^{**}. Recall that the accept/reject reservation function ϕ traces the minimum of ϕ^- and ϕ^+. For a given w_1, if a wage offer w_2 falls below this curve, it is rejected by the couple. Second, the quitting

[7]Note that, from its definition, $\hat{w}$ is the first intersection point between ϕ^- and ϕ^+. Although we cannot rule out other crossings between $\hat{w}$ and $\tilde{w}$, in a very broad range of simulations we never encountered multiple intersections. Consequently, for clarity of exposition, in what follows we draw all our reservation wage figures under the assumption of a single intersection point, and so $\phi^+ > \phi^-$ for $w_1 < \hat{w}$ and $\phi^+ < \phi^-$ for $w_1 > \hat{w}$. None of our theoretical results relies on the uniqueness of intersection points.

wage q is the mirror image of ϕ^+ with respect to the 45^0–line.[8] If the current spouse's wage w_1 is to the left of q, then the employed spouse quits as the unemployed partner accepts a job. Because a quit is conditional on accepting a job, wage combinations that lie below the 45^0–line are not relevant. Notice that the quitting region is the mirror image of the reject region—indeed, one can interpret a quit as a "rejection" of the current wage w_1. Finally, pairs (w_1, w_2) in the region between $\phi^+$and q imply a transition into dual-worker status.

The two functions ϕ and q intersect on the 45^0–line at $(\hat{w}, \hat{w})$. Thus, at $\hat{w}$, the unemployed spouse of a worker-searcher couple is indifferent between accepting and rejecting an offer and, at the same time, her spouse is indifferent between keeping and quitting his job. To emphasize this feature, we refer to $\hat{w}$ as the (smallest) "double indifference point."[9]

Based on this discussion, it should be clear that characterizing the optimal joint-search strategy involves the following steps: (i) studying the conditions under which $w^{**} < \hat{w}$, a necessary inequality to activate the reservation rule $\phi(w_1) = w_1$; (ii) analyzing the shape of ϕ beyond $\tilde{w}$; and (iii) ranking $\tilde{w}$ and $\hat{w}$ relative to w^*, which is useful for comparing joint-search to single-agent search strategies. Proposition 2 tackles (i). Proposition 3 tackles (ii) and (iii) when utility is in the HARA class.

3.2 Risk Neutrality

We begin by presenting the risk-neutral case, then turn to the results with risk-aversion.

Proposition 1 ***[Risk Neutrality]*** *With risk-neutrality, i.e., $u'' = 0$, the joint-search problem reduces to independent single-agent search problems for the two spouses, with value functions $U = 2V$, $\Omega(w_1) = V + W(w_1)$, and $T(w_1, w_2) = W(w_1) + W(w_2)$. Further, $\phi(w_1) = w^{**} = \hat{w} = \tilde{w} = w^*$.*

Figure 3.2 shows the relevant reservation wage functions in the (w_1, w_2) space. As stated in the proposition, $\phi(w_1)$ is simply the horizontal line at w^{**}. Similarly, the quitting function

[8] The portions of these two functions that are not relevant for a couple's actions are plotted as dashed lines vis-a-vis solid lines for the relevant portions.

[9] Since $\hat{w}$ satisfies both (10) and (11), we have $T(\hat{w}, \phi^+(\hat{w})) = \Omega(\hat{w}) = \Omega(\phi^-(\hat{w}))$. Further, $rT(\hat{w}, \hat{w}) = u(2\hat{w})$, so $\hat{w}$ can be solved from $u(2\hat{w}) = r\Omega(\hat{w})$.

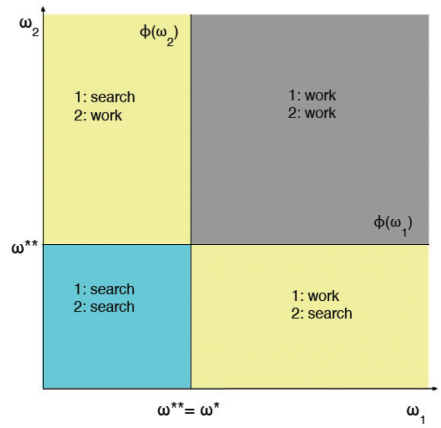


Figure 2: Reservation Wage Functions with Risk Neutrality

$q\,(w_2)$ is the mirror image of $\phi\,(w_1)$ and is shown by the vertical line at $w_1 = w^{**}$. The intersection of these two lines generates four regions, and the couple displays distinct behaviors in each.

3.3 Risk Aversion

To observe deviations between single-agent search and joint-search in this one-location model, risk aversion must be brought to the fore. We start with a key implication of risk aversion summarized by the following proposition.

Proposition 2 ***[Breadwinner Cycle]*** *If u is strictly concave, the reservation wage value of a dual-searcher couple is strictly smaller than the smallest double-indifference point: $w^{**} < \hat{w}$.*

The reservation wage of a dual-searcher couple being strictly smaller than the double-indifference point activates a region where $\phi\,(w_1) = w_1$, which, in turn, gives rise to endogenous quits and to dynamics that we label the "breadwinner cycle" for worker-searcher couples. To understand how this happens, consider Figure 1 for a worker-searcher couple. Suppose that $w_1 \in (w^{**}, \hat{w})$ and the unemployed spouse receives a wage offer $w_2 \in (w_1, \hat{w})$. Because $w_2 > w_1 = \phi\,(w_1)$, the unemployed spouse accepts the offer, which in turn implies that $w_1 < q\,(w_2)$. Since the first spouse's current wage is now below her reservation wage (which just increased), she will quit. As a result, spouses simultaneously switch roles and transit from one worker-searcher couple into another one with a higher wage level. This process might repeat itself over and over again—and the breadwinner alternates—until the employed spouse strictly prefers retaining her job and the pair becomes a dual-worker couple.

3.3.1 HARA utility

To obtain a sharper characterization of the shape of $\phi(w_1)$ beyond $\hat{w}$, we impose more structure on preferences by restricting attention to the HARA (hyperbolic absolute risk aversion) class. Formally, the HARA class is defined as the family of utility functions with linear risk tolerance: $-u'(c)/u''(c) = \rho + \tau c$, where ρ and τ are parameters.[10] This class can be further divided into three subclasses depending on the sign of τ. When $\tau = 0$, absolute risk aversion is independent of consumption level. This is the constant absolute risk aversion (CARA) case, also known as exponential utility: $u(c) = -\rho e^{-c/\rho}$. When $\tau > 0$, absolute risk *tolerance* is increasing with consumption, which is the decreasing absolute risk aversion (DARA) case. A well-known special case of this class is the constant relative risk aversion (CRRA) utility: $u(c) = c^{1-\sigma}/(1-\sigma)$, which obtains when $\rho = 0$ and $\tau = 1/\sigma > 0$. When $\tau < 0$, risk aversion increases with consumption, which is the increasing absolute risk aversion (IARA) case. A special case of this class (for $\tau = -1$) is quadratic utility: $u(c) = -(\rho - c)^2$.

Proposition 3 ***[HARA Utility]*** *With HARA preferences, for $w_1 > \tilde{w}$, the reservation wage function $\phi(w_1)$ and $\tilde{w}$ satisfy:*

$$\phi'(w_1): \begin{cases} >0 \text{ and } \phi(w_1) \leq w_1 & \text{if } u \text{ is DARA} \\ = 0 & \text{if } u \text{ is CARA} \\ <0 & \text{if } u \text{ is IARA,} \end{cases} \qquad \text{and} \qquad \tilde{w}: \begin{cases} > \hat{w} > w^* & \text{if } u \text{ is DARA} \\ = \hat{w} = w^* & \text{if } u \text{ is CARA} \\ = \hat{w} < w^* & \text{if } u \text{ is IARA.} \end{cases}$$

[10] Risk tolerance is defined as the reciprocal of Pratt's measure of "absolute risk aversion." Thus, if risk tolerance is linear, risk aversion is hyperbolic. See Pratt (1964).

Appendix A contains a formal proof of this proposition.[11] It is instructive to sketch the argument behind the proof here. From Lemma 2, we know that beyond $\tilde{w}$ it is never optimal to exercise the quit option and $\phi = \phi^{+}$. Therefore, in this wage range, equation (6) simplifies to:

$$r\Omega\left(w_1\right) = u\left(w_1 + b\right) + \alpha \int_{\phi(w_1)} \left[T\left(w_1, w_2\right) - \Omega\left(w_1\right)\right] dF\left(w_2\right).$$

Substituting out T and Ω by using equations (4) and (10) yields:

$$u\left(w_1 + \phi\left(w_1\right)\right) - u\left(w_1 + b\right) = \frac{\alpha}{r} \int_{\phi(w_1)} \left[u\left(w_1 + w_2\right) - u\left(w_1 + \phi\left(w_1\right)\right)\right] dF\left(w_2\right). \tag{14}$$

Dividing both sides by the left-hand side, we arrive at:

$$1 = \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u\left(w_1 + w_2\right) - u\left(w_1 + \phi\left(w_1\right)\right)}{u\left(w_1 + \phi\left(w_1\right)\right) - u\left(w_1 + b\right)}\right] dF\left(w_2\right). \tag{15}$$

Next, applying a well-known property of HARA preferences established by Pratt (1964, Theorem 1), it can be shown that the RHS of (15) is strictly increasing (decreasing) in w_1 in the DARA (IARA) case, and independent of w_1 in the CARA case. Also, note that the RHS is strictly decreasing in $\phi\left(w_1\right)$. Hence, for the equality to hold in equation (15), in the wage range beyond $\tilde{w}$, $\phi\left(w_1\right)$ must be strictly increasing (decreasing) with DARA (IARA) preferences, and constant with CARA preferences.

CARA case. The left panel of Figure 3 provides a visual summary of the contents of this proposition for the CARA case. The reason ϕ is constant and equal to w^{*} beyond $\hat{w}$ is that, with CARA utility, attitudes toward risk do not depend on the consumption (and hence wage) level. As the wage of the employed spouse increases, the couple's absolute risk aversion remains

[11] It is useful to ask why it is the absolute risk aversion that determines the properties of joint-search behavior, as opposed to, for example, relative risk aversion. The reason is that individuals are drawing wage offers from the same probability distribution regardless of the current wage earnings of the couple. As a result, the uncertainty they face—determined by the dispersion in the wage offer distribution—is fixed, making the attitudes of a couple toward a fixed amount of risk—and therefore, absolute risk aversion—the relevant measure.

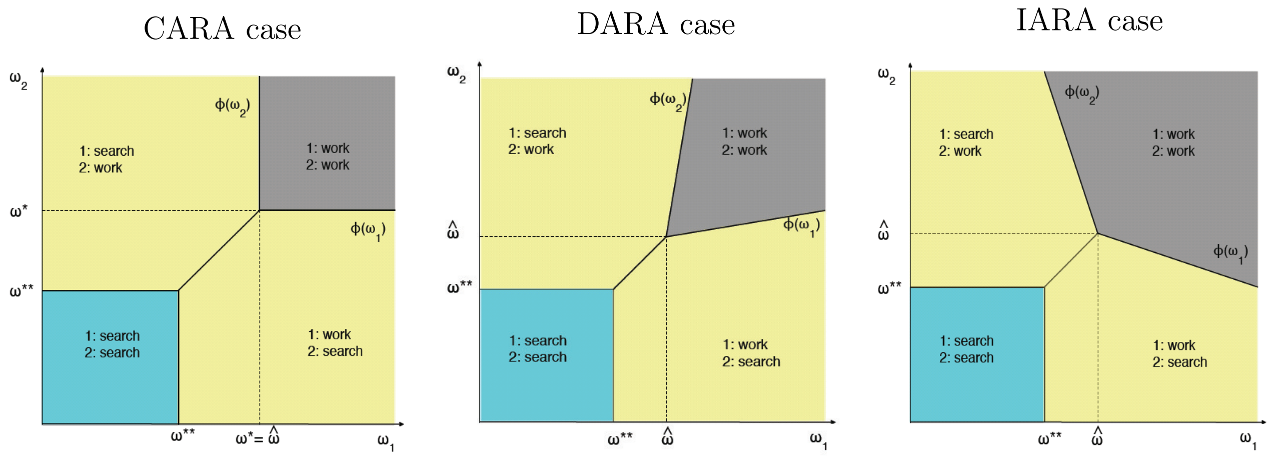


Figure 3: Reservation Wage Functions for HARA–Class Preferences

unaffected, implying a constant reservation wage for the unemployed partner.

Combining the results of Propositions 2 and 3, we conclude that, with CARA preferences, the dual searcher couple is less choosy than the single agent $(w^{**} < w^{*})$. With risk aversion, the optimal search strategy involves a trade-off between lifetime income maximization and the desire for consumption smoothing. Income maximization pushes up the reservation wage, whereas consumption insurance pulls it down since risk-averse agents particularly dislike the low income state (unemployment). The dual-searcher couple can use income pooling to its advantage: it initially accepts a lower wage offer (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income) which remains available to the unemployed spouse. In contrast, when the single agent accepts his job he gives up the search option for good, which induces him to be more picky at the start. Notice that joint search plays a role similar to on-the-job search in its absence, precisely through the breadwinner cycle. We return to this analogy in Section 3.5.

DARA and IARA cases. Under DARA (IARA) preferences, ϕ is increasing (decreasing) with w_1 beyond $\tilde{w}$ (Figure 3, center and right panels). With DARA, a couple becomes less concerned about smoothing consumption as household resources increase and, consequently, becomes more picky in its job search (and vice versa in the IARA case).

An important feature of DARA—one that complicates the proof of Proposition 3—is that breadwinner cycles emerge over a wider range of wages of the employed spouse compared to the CARA and IARA cases. As seen in the center panel of Figure 3, ϕ is strictly increasing in w_1. As a result, there is a wage range where, even when $w_1 > \hat{w}$, a high wage offer may trigger a breadwinner cycle.[12,13]

3.4 Exogenous Separations

We now extend the model to allow for exogenous separations at rate δ. The modifications to the value functions are straightforward, so we omit them (see Appendix A). Under risk neutrality, once again, joint-search collapses to single-agent search. The following proposition characterizes reservation wage strategies in the CARA and DARA case.[14]

Proposition 4 ***[CARA or DARA Preferences with Exogenous Separations]*** *With CARA or DARA preferences, and exogenous job separation, the search behavior of a couple can be characterized as follows:*

(i) There exists a wage $\tilde{w} \in (\hat{w}, \bar{w})$ such that, for any $w_1 > \tilde{w}$, there are no quits.

(ii) For $w_1 \leq \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 > \tilde{w}$, $\phi(w_1)$ is strictly increasing with $\phi(w_1) < w_1$.

(iii) $w^{**} < \hat{w} < \tilde{w}$, which implies that the breadwinner cycle exists.

For DARA preferences, the existence of exogenous separations has *qualitatively* no effect on joint-search behavior, as can be seen by comparing Propositions 3 and 4. However, for CARA

[12] We did not provide a full characterization of the slope of $\phi(w_1)$ in the region between $\hat{w}$ and $\tilde{w}$ for the DARA case. However, in a very broad range of simulations, we never encountered a case where $\phi(w_1)$ was not strictly increasing in that wage range.

[13] In the DARA case, it does not seem possible to rank w^{**} and w^* unless we make further restrictive assumptions. It may seem surprising that we cannot do this by combining Propositions 2 and 3. After all, the logic used to explain why $w^{**} < w^*$ in the CARA case is based on the relative strength of consumption smoothing and income maximization motives. But the argument is more subtle. To see why, consider the one-period gain when deciding whether to accept or reject an offer w. The couple compares $u(b+w)$ to $u(2b)$, whereas the single agent compares $u(w)$ to $u(b)$. The couple makes this comparison at a higher level of consumption and, because of DARA, the couple is less risk averse. This force tends to push w^{**} above w^* and does not allow a general ranking. We verified, by simulation, that with CRRA utility there are parameter configurations where $w^{**} > w^*$.

[14] With separation risk, assets can be used to smooth consumption when agents lose their jobs. This consideration introduces a precautionary saving motive. Thus, as explained earlier, the results in this section do hinge on the no-saving assumption, whereas previous propositions did not require this assumption.

preferences $\phi(w_1)$ is no longer constant beyond $\hat{w}$: it increases with w_1. In the context of joint-search, the separation risk has two effects. Consider the problem of the worker-searcher couple with wage w_1 contemplating an offer w_2. First, there is the risk associated with the duration of the new job offered to the searching spouse. Second, there is the risk of job loss for the currently employed spouse.

The first effect of exogenous separations is also present in the single-agent search model: if the expected duration of a job is lower (high δ), the unemployed agent reduces her reservation wage for all values of w_1. The higher is w_1, the smaller this effect is, since the marginal utility from the additional income decreases in w_1. Since, under CARA/DARA utility, $\phi(w_1)$ is weakly increasing when $\delta = 0$, with $\delta > 0$ the function $\phi(w_1)$ becomes strictly increasing.

The second effect is related to the event that the currently employed spouse might lose his job. If the couple turns down the offer at hand and the job loss indeed occurs, its income will fall from $w_1 + b$ to $2b$ for a net change of $b - w_1 < 0$. Clearly, this income loss (and, therefore, the fall in consumption) increases with w_1. If instead the couple accepts the job offer and spouse 1 loses his job, income will change from $w_1 + b$ to $b + w_2$, for a net change of $w_2 - w_1$. On the one hand, setting the reservation wage to $\phi(w_1) = w_1$ would completely insure the downside risk of spouse 1 losing his job (because then $w_2 - w_1 \geq 0$). At the same time, letting the reservation wage rise this quickly with w_1 reduces the probability of an acceptable offer and increases the probability that the searcher will still be unemployed when spouse 1 loses his job. The optimal search strategy balances these two forces by letting $\phi(w_1)$ rise with w_1, but less than one for one.[15]

From this discussion, it should be clear that one cannot prove a general result on the slope of ϕ beyond $\hat{w}$ in the IARA case with exogenous separations. On the one hand, the economic forces associated with job destruction risk make ϕ an increasing function of w_1. On the other hand, IARA pushes the reservation wage down as w_1 increases.

[15]This mechanism is closely related to Lise (2010), in which individuals climb the wage ladder but fall to the same unemployment benefit level upon layoff. As a result, in his model, the savings rate increases with the current wage, whereas this precautionary demand manifests itself as delayed offer acceptance in our model.

3.5 Additional Results

We now state some additional results of optimal joint search.

Consumption as a Private Good within the Couple. In the baseline model, we have assumed that goods consumed by the household are public within the couple. We now take the polar view that consumption is a private good for the couple. In keeping with the symmetry assumption adopted throughout the paper, we impose that the two spouses have the same weight in household utility, and hence per-capita intra-period household utility is $u\left(\frac{y_1+y_2}{2}\right)$. One can easily adapt all the proofs and show that all the results stated so far are still true, the only exceptions being that in the CARA case we have $w^* < \hat{w}$, and in the DARA and IARA cases this ranking becomes ambiguous. See Appendix B for details. We conclude that our findings are largely independent of the degree to which consumption is private within the household.

Equivalence with Single Agent Search. Besides risk neutrality, there are two other important cases where joint-search strategies are equivalent to those of a single agent, as we formally prove in Appendix B.

The first case is when couples with CARA utility are free to save and borrow, and debt constraints do not bind. Borrowing effectively substitutes for the consumption smoothing provided within the household through interdependent job search strategies, making the latter redundant.

The second case is when couples can search with the same effectiveness on and off the job. Through the breadwinner cycle, joint search offers the couple a way to climb the wage ladder: one can view joint search as a costly version of on-the-job search. The cost comes from the fact that, absent on the job search, in order to keep the search option active, the pair must remain a worker-searcher couple and forgo the full wage earnings of a dual-worker couple. When on-the-job search is explicitly introduced and the offer arrival rate is equal across employment states, it completely neutralizes the benefits of joint search.

An Isomorphism: Search with Multiple Job Holdings. The joint-search framework analyzed so far is isomorphic to a search model with a single agent who can hold multiple jobs at the same time. To see this, suppose that the time endowment of a worker can be divided into two subperiods (e.g., day shift and night shift). The single agent can be (i) unemployed and searching for his first job while enjoying $2b$ units of home production; (ii) working one job at wage w_1 while searching for a second one; or (iii) holding two jobs with wages w_1 and w_2. It is easy to see that the problem faced by this individual is exactly given by the equations (4), (5), and (6) and therefore it has the same solution as the joint-search problem.[16] Consequently, for example, when the agent works in one job and gets a second job offer with a sufficiently high wage, he will accept the offer and simultaneously quit the first job to search for a better one. Here, it is not the breadwinners who alternate, but the jobs that the individual holds.

4 Quantitative Analysis

The goal of this section is twofold. First, we calibrate the model to match basic facts about the US labor market and present some illustrative simulations to gain some sense about the quantitative differences in labor market outcomes between single- and joint-search economies. For example, a priori it is not obvious whether the joint-search economy would have a higher or lower unemployment rate: for dual-searcher couples, w^{**} is below w^*, but for worker-searcher couples $\phi(w_1)$ may be above w^* for a wide range of values of w_1. Second, we turn to US micro data from the Survey of Income and Program Participation (SIPP) and show that some key implications of the simulated model are quantitatively in line with the corresponding stylized facts about job search behavior of couples.

[16] There is a further implicit assumption here: the arrival rate of job offers is proportional to the nonworking time of the agent (that is, 2α when unemployed and α when working one job).

4.1 Model Simulations

We focus on the model with CRRA utility and exogenous job terminations. We first simulate labor market histories for a large number of "singles." We then pair these "singles" together to form couples that conduct joint search in the same economy (i.e., under the same set of parameters). We use the same sequence of wage offers and separation shocks for each individual in both economies, and compare some key labor markets statistics (e.g., mean wage, unemployment rate, unemployment duration, separation rate, etc.) across economies.

Calibration. We calibrate the economy with singles to replicate some salient features of the US economy. The time period in the model is set to one week. The economy is characterized by the following set of parameters: $\{\rho, r, F, \delta, \alpha, b\}$. The coefficient of relative risk aversion, ρ, varies from zero (risk neutrality) to eight in simulations. The weekly net interest rate, r, is set equal to 0.001, corresponding to an annual interest rate of 5.3%. The wage offer distribution F is a truncated log-normal with standard deviation $\sigma = 0.1$, mean $\mu = -\sigma^2/2$ (so that the average wage offer is normalized to one), and truncation point at three standard deviations above the average. We set $\delta = 0.0054$ to reproduce a monthly exogenous separation rate of 2%. For each risk aversion value, the offer arrival rate, α, is recalibrated to generate an unemployment rate of 5.5%.[17] Finally, the value of leisure, b, is set to 40% of the mean of the wage offer distribution.

Results. Table I reports some key statistics of the two economies. The first two columns confirm Proposition 1: under risk neutrality ($\rho = 0$) the joint-search economy coincides with the single-agent search economy. Next, consider the case $\rho = 2$. The reservation wage of the dual-searcher couple is 23% lower than in the single-agent search economy, which is reflected in the shorter unemployment durations for these couples. At the same time, though, the reservation wage of worker-searcher couples is higher than w^* for a wide range of the employed spouse's

[17] As ρ goes up, w^{**} falls and unemployment duration decreases. So, to continue matching an unemployment rate of 5.5%, we need a lower value of α. For example, for $\rho = 0$, $\alpha = 0.25$ and for $\rho = 8$, $\alpha = 0.097$.

Table I: Single versus Joint Search: CRRA Preferences and Exogenous Separations

	$\rho = 0$		$\rho = 2$		$\rho = 4$		$\rho = 8$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
Reservation wage (w^* or w^{**})	1.01	1.01	0.96	0.74	0.80	0.58	0.59	0.48
Reservation wage $\phi(1)$	–	n/a	–	1.00	–	0.93	–	0.88
Double indifference point $\hat{w}$	–	1.01	–	1.00	–	0.93	–	0.80
Mean wage among employed	1.05	1.05	1.05	1.09	1.00	1.05	1.00	1.01
Mean-min wage ratio	1.04	1.04	1.09	1.47	1.25	1.81	1.70	2.10
Unemployment rate	5.5%	5.5%	5.5%	7.7%	5.5%	7.5%	5.5%	5.7%
Unemployment duration	10.8	10.8	10.8	14.5	10.8	14.4	10.8	11.2
Dual-searcher	–	5.8	–	3.9	–	5.7	–	5.9
Worker-searcher	–	10.3	–	14.0	–	13.7	–	10.6
Quits/Separations	–	0%	–	5.9%	–	3.3%	–	0.5%
Breadw. cycles/UE transitions	–	0%	–	7.4%	–	4.3%	–	1.0%
Welfare gain (consumption)	–	0%	–	4.7%	–	14.7%	–	25.5%
Welfare gain (income)	–	0%	–	1.2%	–	2.6%	–	0.5%

wage.[18] For example, for every wage above the mean of the wage offer distribution (equal to one), the reservation function ϕ is above w^*, implying a longer unemployment duration than for singles. Overall, this second effect dominates and the joint-search economy displays higher average unemployment duration—14.5 weeks instead of 10.8—and higher unemployment rate, 7.7% instead of 5.5%.[19]

Comparing mean wages tells a similar story. The job-search choosiness of worker-searcher couples dominates the insurance motive of dual-searcher couples, so the average wage for individuals in couples is higher than for singles.

The endogenous quit rate (a reflection of the breadwinner cycle in action) is sizable: 5.9% of all separations are quits, and 7.4% of all workers making unemployment to employment

[18] Two further findings that hold true for all the parameterizations reported in Table I are that (i) $\tilde{w}$ is only slightly higher than $\hat{w}$, and (ii) between these two points we have: $0 < \phi'(w_1) < 1$.

[19] To compute the average unemployment duration by type of couple, we used the following definitions. An unemployment spell of a dual-searcher couple starts the first week both spouses are unemployed, and ends the week when one of the unemployed spouses accepts a job offer (a transition into worker-searcher couple). An unemployment spell of a worker-searcher couple starts the first week in which one spouse is unemployed and the other is employed. It ends when either the job of the employed spouse exogenously terminates (a transition into dual-searcher couple) or when the unemployed spouse accepts a job offer –independently of whether the employed spouse quits. If she does quit, a new unemployment spell of a worker-searcher couple begins.

(UE) transition have partners making the opposite transition in the same week.[20]

The next four columns in Table I display how these statistics change as we increase risk aversion. As ρ increases, both w^* and w^{**} fall because of the stronger demand for consumption smoothing that makes agents accept job offers more quickly. Notice, however, that the gap between the two first widens and then shrinks. This is intuitive: as $\rho \to \infty$, it must be true that $w^* = w^{**} = b$, so the two economies converge again. As for $\phi(1)$, it falls as risk aversion increases, implying that worker-searcher couples accept job offers more quickly, thus reducing their unemployment duration and the frequency of breadwinner cycles.

We also report a measure of frictional wage dispersion, the mean-min ratio (Mm), defined as the ratio between the mean wage and the lowest wage, i.e., the reservation wage. Hornstein, Krusell, and Violante (2009) demonstrate that the single-agent search model with homogeneous workers, when plausibly calibrated, generates very little frictional wage dispersion.[21] What is novel here is that the joint-search model can generate more frictional dispersion: the reservation wage for the dual-searcher couple is lower (which translates into a lower minimum wage) and the couple can climb the wage ladder faster (which translates into a higher mean wage).

Finally, we present two separate measures of the welfare gains of joint search. Recall that couples have two advantages over singles: first, they can smooth consumption better; second, they can attain a higher lifetime earnings. The first measure of welfare gain is the standard consumption-equivalent variation and captures both benefits of joint-search.[22] Not surprisingly, given the absence of saving, the welfare gain by this measure is very large and increases with risk aversion (ranging from 4.7% to 25.5% of lifetime consumption). The second measure is the increase in lifetime income that is due to joint-search and isolates the effect of

[20]The reason why these two fractions are not the same is that, in the (discrete time) simulations, it is possible that during the same week when an unemployed spouse in a worker-searcher couple finds a job, his employed partner's job is terminated. Such a transition would be indistinguishable from a "true" breadwinner-cycle in any micro data set (including the SIPP which we use in the next section), and hence it would be counted as such. For this reason, we also include these episodes as "measured" breadwinner cycles.

[21]The fifth row of Table I confirms this result. It also confirms the finding in Hornstein et al. that the Mm ratio increases with risk aversion.

[22]To make the welfare comparison between singles and couples meaningful, we assume that consumption is a private good (as in Section 3.5), so each spouse consumes half of household income.

the breadwinner cycle. This effect can also be quite large: for example, the gain in lifetime income is roughly 2.6% when $\rho = 4$.

4.2 Stylized Facts on Joint Search: A First Look

This section investigates whether some of the key predictions of the simulated joint-search model are borne out by the micro data.

4.2.1 Data and Sample Selection

Data. Our empirical analysis is based on micro data from the Survey of Income and Program Participation (SIPP). We use the 1996 panel, which contains twelve waves (48 months of data) starting in December 1995—an ideal period for our analysis because of the stationary aggregate labor market conditions.

The SIPP has several features that make it an ideal data set for our purposes. First, it is a longitudinal survey, essential to our investigation. Second, one aim of the SIPP is measuring worker turnover. Therefore, the problem of classification error is presumably much less severe than in other data sets. In particular, it contains *weekly* labor-force status information which makes the measurement of transitions very precise.[23] Third, a full employment, earnings and benefits history is available for all household members.[24]

Sample selection. We construct a sample of individuals with strong labor force attachment who are likely to engage in job search when out of work (e.g., we exclude individuals if they are enrolled in school). Our analysis of Section 3.5 suggests an "equivalence" between single-agent and joint search for households with sizable savings and with occupations where on-the-job search is very effective. Therefore, deviations from single-agent search behavior in the data

[23] At least since Gottschalk and Moffitt (1999), the SIPP has become a standard source to study labor market transitions. The greatest measurement challenge in the SIPP is the seam bias: a disproportionate number of labor-market transitions are reported as taking place between waves, not during waves. We did find some evidence of this pattern in our sample, in that there was a spike in the frequency of spells of 17 weeks. However, we verify that our results are robust to the exclusion of those observations.

[24] For our investigation, this latter feature is a distinct advantage over the Panel Study of Income Dynamics (PSID), which follows in detail only heads of households. See also Dey and Flinn (2008) for a similar motivation.

are more likely to be detectable among young and low-educated households.[25] In light of this, we restrict our sample to individuals aged 20-40. We report results both for workers of all education levels and for workers with at most a high-school diploma. Our final sample comprises 335 unemployment spells for singles and 645 for couples. Appendix C contains more details on sample selection and a table with descriptive statistics for our final sample.

4.2.2 Findings

We now document some stylized facts of joint-search behavior and investigate whether they are quantitatively consistent with the simulated model of Section 4.1.

First, we are interested in how the employed spouse's wage affects the job offer acceptance decision of worker-searcher couples. To this end, we regress log unemployment duration of worker-searcher couples on the log wage of the employed spouse. The estimated elasticity is 0.33 (S.E. 0.07): doubling the wage of the employed spouse increases unemployment duration of the unemployed partner by a third. This finding is qualitatively consistent with the joint-search model with CRRA utility and exogenous separations, where the transition between worker-searcher status and dual-worker status is regulated by a reservation wage function that increases with the employed spouse wage. Running the same regression on simulated data from the model of Section 4.1 yields elasticities in the range 0.1-0.5 as we vary ρ from one to eight. For ρ around two, the elasticity is around 0.3, as estimated in the micro data.

Next, we analyze mean unemployment duration by household type. Table II reports the results. Worker-searcher couples have the longest spells (14 weeks), followed by singles (12 weeks) and, finally, by dual searcher couples who have much shorter spells of job-search on average (7 weeks).[26] Differences across household types are always statistically significant. Excluding households with high education levels yields similar results—only durations for dual searcher are somewhat shorter (5 weeks). These facts about unemployment durations line up

[25]It is well known that wealth increases steeply with education level and with age until retirement. Table 4 in Nagypal (2008) shows that the importance of job-to-job transitions, as a fraction of total separations, increases with age and education.

[26]The definition of an unemployment spell for dual searcher couples and worker-searcher couples is the same used in the simulations of Section 4.1.

Table II: Unemployment Duration by Household Type

	All Education	Less than H-S
Worker-Searcher Couple	13.59*	15.33*
	(0.46)	(0.76)
Dual Searcher Couple	6.61*	4.56*
	(1.14)	(0.62)
Single	11.84*	11.85*
	(0.51)	(0.82)
N of obs.	980	412
	Hypothesis Testing $(\Pr > \chi^2)$	
Worker-Searcher = Single	0.009	0.002
Worker-Searcher = Dual Searcher	0.000	0.000
Dual Searcher = Single	0.000	0.000

Note: Bootstrapped S.E. based on 500 replications. * denotes significance at 1 percent.

closely with the predictions of the calibrated model in Table I. In the range between $\rho = 2$ and $\rho = 8$, the average duration for worker-searcher couples varies between 11 and 14 weeks, for dual searcher couples it varies between 4 and 6 weeks, and for singles it always equals 10.8 weeks by construction. Overall, the differences in job search durations across household types implied by the model are very close to those estimated in our SIPP sample.

We now explore the presence of breadwinner cycles in the data. We define a breadwinner cycle as a worker-searcher to searcher-worker (or vice versa) transition with a possible intervening dual worker spell of at most 4 weeks.[27] We find that 7.6% of all the transitions from unemployment to employment (UE) for individuals in couples involve a breadwinner cycle. Recall that, in simulations, this fraction rises from 1% when $\rho = 8$ to 7.4% when $\rho = 2$, suggesting again that the data are closest to the model with risk aversion around two.

Finally, we acknowledge that since the simulated sample is much larger and longer than the SIPP sample, one may be concerned that our results are affected by small sample bias and right-censoring bias in the data. We therefore recomputed all our statistics on an artificial sample with the same number of spells for singles and couples, and the same length (192 weeks) as our empirical sample. The results were largely unchanged.[28]

[27] In the data, about half of these cycles occurs within one week.

[28] For $\rho = 2$, the findings are as follows. First, the elasticity of unemployment duration to the couple's

5 Joint Search with Multiple Locations

The importance of the geographical dimension of job search is undeniable. For a single agent, accepting a job in a different location could require a moving cost high enough to induce her to turn down the offer. For a couple, this spatial dimension introduces an additional friction with important ramifications for joint job search. A couple is likely to suffer from the disutility of living apart if spouses work in different locations. This cost of living apart can easily rival the physical cost of relocation, since it is a flow cost as opposed to the latter, which is arguably better thought of as a one-time cost.

The introduction of location choice leads to important changes in the search behavior of couples compared to a single agent, *even* with risk neutrality. To make this comparison sharper, we focus precisely on the risk-neutral case. Furthermore, many of these changes are not favorable to couples. As a result, joint search can create new frictions as opposed to the new opportunities studied in the one-location model.[29]

To keep the analysis tractable, we first consider agents searching for jobs in two symmetric locations and provide a theoretical characterization of the solution. Then, we examine the more general case with $L(>2)$ locations that is more suitable for a meaningful calibration, and provide some results based on numerical simulations.

5.1 Two Locations

Environment. The economy has two locations and individuals are risk neutral. Couples incur a flow resource cost, denoted by κ, if the spouses live apart. Denote by i the "inside" location, i.e., the location where the couple resides, and by o the "outside" location. Unem-

wage for worker-searcher couples is 0.29. Second, the average duration for worker-searcher couples is 14.4 weeks, for dual searchers is 3.1 weeks, and for singles is 11.1 weeks. Third, the fraction of UE transitions involving breadwinner cycles is 6.5%. In sum, discrepancies are minor. Simulation results show that such small discrepancies are mostly due to the small sample size as opposed to the right-censoring. Intuitively, the empirical sample is quite long relative to the mean length of jobless spells.

[29]This friction raises the issue of whether, in some states, the couple should split. While the interaction between labor market frictions and changes in marital status is a fascinating question, it is beyond the scope of this paper. Here we assume that the couple has committed to stay together or, equivalently, that there is enough idiosyncratic non-monetary value in the marriage to justify continuing the relationship.

ployed individuals receive job offers at rates α_i and α_o, respectively, from the inside and outside locations. Both locations have the same wage offer distribution, F. We assume away moving costs: the aim of the analysis is the comparison with the single-agent problem, and such costs would also be borne by the single agent.

A couple can be in one of four labor market states. In addition to the dual-searcher and worker-searcher couples, now couples can have two different dual-worker statuses. If both spouses are employed in the same location we refer to them as a "dual-worker couple" with value function $T(w_1, w_2)$; if they are employed in different locations we refer to them as a "*separate* dual-worker couple" (another absorbing state) with value function $S(w_1, w_2)$.[30] The corresponding value functions are:

$$rT(w_1, w_2) = w_1 + w_2 \tag{16}$$

$$rS(w_1, w_2) = w_1 + w_2 - \kappa \tag{17}$$

$$rU = 2b + 2(\alpha_i + \alpha_o) \int \max\{\Omega(w) - U, 0\} \, dF(w) \tag{18}$$

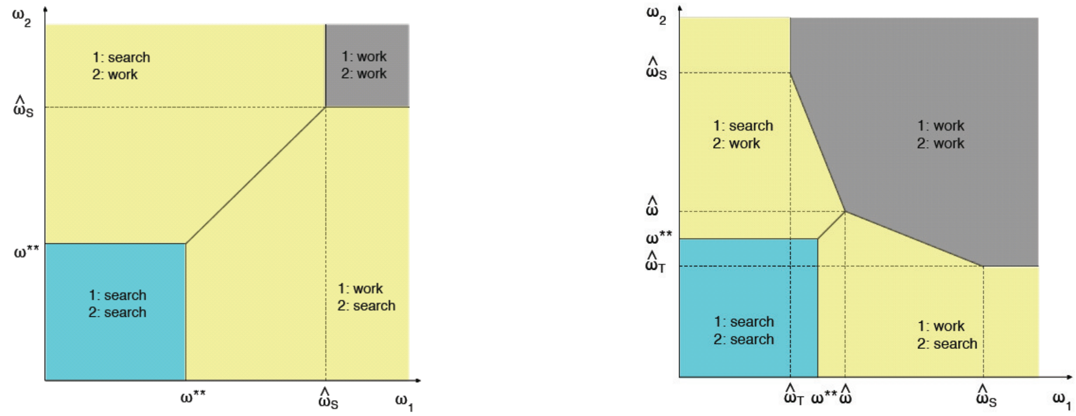
$$\begin{aligned} r\Omega(w_1) = {} & w_1 + b + \alpha_i \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} \, dF(w_2) \\ & + \alpha_o \int \max\{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} \, dF(w_2). \end{aligned} \tag{19}$$

The first three equations are easily understood, and the definition of $\Omega(w_1)$ now has to account separately for inside and outside offers. The decision of the dual-searcher couple is entirely characterized by the reservation wage w^{**}. For a worker-searcher couple, let $\phi_i(w_1)$ and $\phi_o(w_1)$ be the reservation functions corresponding to inside and outside offers. Once again, these functions are piecewise with one piece corresponding to the 45^0–line. As in the one-location case, the same functions $\phi_i(w_2)$ and $\phi_o(w_2)$ characterize the quitting decision.

It is easy to see that the single-agent search problem with two locations is the same as the one-location case (with the arrival rate, α, in equation (3) replaced by $\alpha_i + \alpha_o$). We keep

[30] Because of symmetry across locations, couples with a searching spouse have no advantage from living separately.

Figure 4: Reservation Wage Functions for Outside (Left) and Inside (Right) Offers



labeling the single-agent reservation wage w^*. The next proposition characterizes the optimal joint-search strategies in the two-location case, whenever there is a positive cost κ of living apart.

Proposition 5 ***[Two Locations]*** *With two locations, risk neutrality, and $\kappa > 0$, the search behavior of a couple can be characterized as follows. There is a wage value*

$$\hat{w}_S = b + \kappa + \frac{\alpha_i}{r} \int_{\hat{w}_S - \kappa} [1 - F(w)]\, dw + \frac{\alpha_o}{r} \int_{\hat{w}_S} [1 - F(w)]\, dw$$

and a corresponding value $\hat{w}_T = \hat{w}_S - \kappa$ such that:

(i) [Outside offers]: for $w_1 < \hat{w}_S$, $\phi_o(w_1) = w_1$, and for $w_1 \geq \hat{w}_S$, $\phi_o(w_1) = \hat{w}_S$.

(ii) [Inside offers]: for $w_1 \leq \hat{w}$, $\phi_i(w_1) = w_1$, for $w_1 \in (\hat{w}, \hat{w}_S)$, $\phi_i(w_1)$ is strictly decreasing, and for $w_1 \geq \hat{w}_S$, $\phi_i(w_1) = \hat{w}_T$.

(iii) $w^{**} \in (\hat{w}_T, \hat{w})$, whereas $w^* \in (\hat{w}, \hat{w}_S)$. Since $w^{**} < \hat{w}$, the breadwinner cycle exists.

The first result is that a dual-searcher couple is less choosy than a single agent because it is effectively facing a worse offer distribution: some wage configurations are attainable only in separate locations, hence by paying the cost κ.

Figure 4 shows the reservation functions for both outside and inside offers. Consider outside offers (left panel) to the unemployed spouse of a worker-searcher couple where the employed spouse earns $w_1 < \hat{w}_S$. Any wage less than w_1 is rejected. For offers exceeding w_1, the employed worker quits his job and follows his spouse to the outside location: his earnings are not high enough to justify living apart and paying κ. In this region, the breadwinner cycle is active *across locations*. In contrast, when $w_1 > \hat{w}_S$ and the couple receives a wage offer $w_2 > \hat{w}_S$, it will bear the cost of living separately in order to keep both of those high wages.

Comparing the two panels of Figure 4, it is immediate that inside offers are accepted by a worker-searcher couple over a broader range (of w_1 values), since no cost κ has to be paid. The function $\phi_i(w_1)$ has three distinct pieces. For w_1 small enough, $\phi_i(w_1) = w_1$, and the breadwinner cycle is active. For w_1 large enough, it is constant. A new intermediate range $(\hat{w}, \hat{w}_S)$ arises where the function is decreasing. This is because ϕ_o is increasing in this range: as w_1 rises, the expected gains from search accruing through outside offers are lower (it takes a higher outside wage offer w_2 to induce the employed spouse to quit) and the reservation wage for inside offers falls.

The multiple-location model *with risk neutrality* shares two results with the one-location model *with risk aversion*: (i) the unemployed couple being less picky than the individual, and (ii) the breadwinner cycle. However, the economic mechanisms are different in the two models.

Tied-movers and tied-stayers. In a seminal paper, Mincer (1978) studied empirically the job-related migration decisions of couples in the United States. Following the terminology introduced by Mincer, we refer to a spouse who rejects an outside offer that she would accept when single as a "tied-stayer." Similarly, we refer to a spouse who follows her partner to the new destination even though her individual calculus (as single) would dictate otherwise as a "tied-mover."[31]

[31] Using data from the 1962 Bureau of Labor Statistics (BLS) survey of unemployed persons, Mincer (1978) defined an individual to be a tied-stayer (a tied-mover) if the individual cites his/her spouse's job as the main reason for turning down (accepting) a job in a different location: Mincer wrote (page 758): "The unemployed were asked whether they would accept a job in another area comparable with the one they lost. A positive answer was given by 30 percent of the married men, 21 percent of the single women, and only 8 percent of the married women. Most people who said no cited family, home, and relatives as reasons for the reluctance to move. However, one quarter of the women singled out their husbands' job in the present area as the major deterrent factor." Overall, Mincer estimated that roughly two-thirds of the wives of moving families are tied-movers, and over one third of wives in families of stayers are tied-stayers.

Figure 5: Tied-Stayers and Tied-Movers in the Joint-Search Model

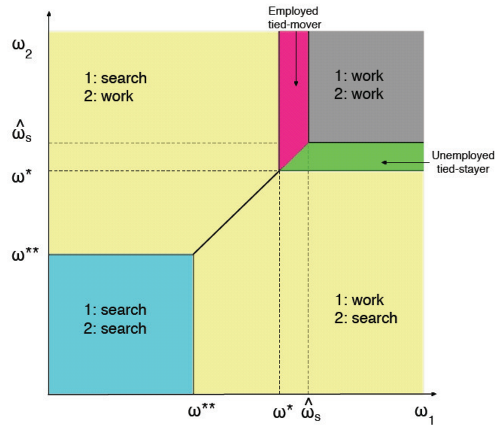


Figure 5 redraws $\phi_i(w_1)$ and indicates the regions giving rise to tied-stayers and tied-movers in our model. When $w_1 > w^*$, the unemployed spouse rejects all outside offers with $w_2 \leq \phi_i(w_1)$ and stays in the current location. In contrast, a single agent would accept all offers w_2, which is less than $\phi_i(w_1)$ by Proposition 5. Therefore, an unemployed spouse who rejects an outside wage offer $w_2 \in (w^*, \phi_i(w_1))$ is formally a tied-stayer. Second, there is a region in which the currently *employed* spouse becomes a tied-mover. Suppose that $w_1 \in (w^*, \hat{w}_S)$. If the unemployed spouse receives an outside offer higher than w_1, she will accept it, the employed spouse will quit his job, and both will move to the other location. In contrast, if the employed spouse were single, she would not have moved to the outside location because she would not even be searching for a job. Thus, the employed spouse is a tied-mover.

Both sets of choices involve potentially large concessions by a spouse compared to the situation where he/she were single, but they are optimal from a joint-search perspective. This feature opens the possibility of welfare costs of joint search, an aspect of the model that we analyze quantitatively, through simulation, in the next section.

Finally, we note that the isomorphism to the single-agent search model with multiple job holding extends to this set up as well. It is enough to think of κ as a commuting cost the agent would incur when holding two jobs in different locations.

Table III: Single versus Joint Search: Multiple Locations

	Cost of Living Separately Per Spouse			
	$\kappa = 0.0$		$\kappa = 0.1$	$\kappa = 0.3$
	Single	Joint	Joint	Joint
Reservation wage (w^* or w^{**})	1.01	1.01	0.96	0.90
$\hat{w}_T$	–	1.01	0.94	0.84
Double indiff. point ($\hat{w}$)	–	1.01	0.98	0.94
$\hat{w}_S$	–	1.01	1.03	1.10
Reservation wage ϕ_i (1)	–	n/a	0.96	0.90
Mean wage	1.05	1.05	1.05	1.05
Mean-min wage ratio	1.04	1.04	1.12	1.25
Unemployment rate	5.5%	5.5%	6.7%	15.6%
Unemployment duration	10.8	10.8	11.4	17.6
Dual-searcher	–	5.8	3.1	2.6
Worker-searcher	–	10.3	11.1	17.3
Movers (% of population)	0.5%	0.5%	0.6%	1.1%
Stayers (% of population)	0.6%	0.6%	0.9%	2.6%
Tied-movers/Movers	–	0%	20.0%	51.3%
Tied-stayers/Stayers	–	0%	14.6%	33.1%
Quits/Separations	–	0%	14.1%	44.5%
Welfare gain (income)	–	0%	−0.5%	−6.6%

5.2 Illustrative Simulations

For this simulation exercise, we extend the two-location model to allow for multiple locations and exogenous separations. Specifically, consider an economy with L geographically separate symmetric labor markets. Firms in each location generate offers at flow rate ψ. A fraction θ of total offers are distributed equally to the $L - 1$ outside locations and the remaining $(1 - \theta)$ is made to the local market.[32]

L is set to nine, representing the number of US census divisions; θ is set to $1 - 1/L$, implying that firms make offers to all locations with equal probability. We analyze values of κ between 0 and 0.3. Because this cost is shared between two spouses, $\kappa = 0.3$ corresponds to a flow cost of slightly less than 15% of the average *household* income of a dual worker couple. The remaining parameter values are exactly the same as those used in the simulations of the

[32] The assumption that there is a very large number of individuals in each location, combined with the fact that the environment is stationary (i.e., no location-specific shocks), implies that we can take the number of workers in each location as constant, despite the fact that workers move freely across locations.

one-location model of Section 4.1 for the risk-neutral case ($\rho = 0$).

Table III presents the results. A comparison of the first two columns confirms that the single-agent and joint search problems are equivalent when $\kappa = 0$. The third and fourth columns show the simulation results when $\kappa = 0.1$ and $\kappa = 0.3$. The reservation wages are in line with our theoretical results in Proposition 5: $\hat{w}_T < w^{**} < w^* < \hat{w}_S$. A positive cost κ makes outside offers less appealing, inducing couples to reject some offers that a single would accept. As a result, the unemployment rate is higher under joint search. For example, when $\kappa = 0.3$ the unemployment rate is 15.6% compared to 5.5% in the single-agent model. Average unemployment duration increases from 10.8 weeks to 17.6 weeks as κ rises from 0 to 0.3. The duration for dual-searcher couples is shorter than for single agents (since $w^{**} < w^*$) and gets even shorter as κ increases (falls from 5.8 weeks to 2.6 weeks). However, because worker-searcher couples face a smaller number of feasible job offers from outside locations, they have longer unemployment spells: 11.1 weeks when $\kappa = 0.1$ and 17.3 weeks when $\kappa = 0.3$, compared to 10.8 weeks when $\kappa = 0$. Overall, there are more jobless workers at any point in time, and some of these unemployed individuals—those in worker-searcher families—stay unemployed for much longer than they would have, had they been single.

We next turn to the impact of joint search on the mobility decision of couples. In our context, we call "movers" only those couples who move to another location because one of the spouses accepts an outside job offer.[33] Similarly, we define a couple to be a "stayer" if either member of the couple turns down an outside job offer. Using this definition, the fraction of movers in the population is 0.5% *per week* when $\kappa = 0$; it rises to 1.1% when $\kappa = 0.3$.[34] Notice that while the fraction of movers appears high, this is not surprising given that we assumed away physical costs of moving. Perhaps more strikingly, 51.3% of all movers are tied-movers, using Mincer's definition, when $\kappa = 0.3$. The fraction of tied-stayers is also sizable: 33.1% in

[33]If one of the spouses belonging to a separate dual-worker couple receives a separation shock and becomes unemployed, she will move to her spouse's location. In this case, we do not consider the household a mover, since the move did not occur in order to accept a job.

[34]Part of the rise in the moving rate is mechanically related to the rise in the unemployment rate with κ: because there is no on-the-job search, individuals only get job offers when they are unemployed, which in turn increases the number of individuals who accept offers and move.

the high κ case. The fraction of job separations due to voluntary quits is as large as 44.5% in the high κ case.

Finally, a comparison of lifetime wage incomes shows that the friction introduced by the interaction of joint search and multiple locations can be substantial: lifetime income of a couple is reduced by about 0.5% (per person) compared to a single agent when $\kappa = 0.10$ and by 6.6% when $\kappa = 0.3$. Overall, these results show that with multiple locations, joint-search behavior can differ substantially from the standard single-agent search.

6 Conclusions

Search theory has almost exclusively focused on the single-agent problem, ignoring the ramifications of joint search for labor market dynamics. This paper characterizes theoretically the joint job-search behavior of couples in a variety of economic environments.

As is often the case in theoretical analyses, we had to strike a balance between generality and tractability to make sharp statements about optimal joint-search behavior. Structural empirical analyses of the data may require richer models. However, knowing the properties of the reservation wage functions in special cases (like ours) provides guidance towards the numerical solution and the interpretation of simulation-based results in these more complex joint-search environments. From a theoretical viewpoint, there are additional forces that could influence joint-search decisions in the labor market beyond those studied in this paper. Some examples include complementarity/substitutability of leisure between spouses (Burdett and Mortensen, 1977), or consumption-sharing rules within the family that deviate from full income pooling, as in the collective model (Chiappori, 1992), or the option given to the couple to split and break up the marriage (Aiyagari, Greenwood, and Guner, 2000), or fundamental asymmetries between men and women.

One key challenge in the advancement of this research program is the access to micro data with household-level, high-frequency information on labor market histories of both members of the couple and on their geographical movements. Data in such format would allow a structural estimation of the model.[35] While a full structural estimation is beyond the scope of this paper, we made a first step towards uncovering patterns of joint-search behavior in the micro data. In light of our theoretical results, we conjectured that deviations from single-agent search behavior are more likely to be detectable among young and poor households, who are closer to being hand-to-mouth consumers. Using data from the SIPP, we found that, indeed, among such households the breadwinner cycle appears to be active, and the unemployment durations of different types of households are broadly consistent with the predictions of the joint search model.

Looking ahead, it will be interesting to enrich our environment with an equilibrium determination of the wage distribution and study the conditions under which joint search may offer another resolution to the Diamond paradox, which undermines the standard single-agent equilibrium search model.

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[35] A more feasible task is the structural estimation of a search model to understand patterns of multiple job holding, an environment that we showed to be isomorphic to joint search, under some assumptions. The survey data needed for such a task are more readily available.

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NOT FOR PUBLICATION
Supplemental Appendix

A Proofs

Proof. [Lemma 1] Rewrite equation (6) using equation (4):

$$r\Omega(w) = u(w+b) + \alpha \int \max\left\{\frac{u(w+w_2)}{r} - \Omega(w), \Omega(w_2) - \Omega(w), 0\right\} dF(w_2). \tag{A1}$$

Add $\alpha\Omega(w)$ to both sides of the equation and divide them by $\alpha + r$, to get

$$\Omega(w) = \frac{u(w+b)}{\alpha+r} + \frac{\alpha}{\alpha+r} \int \max\left\{\frac{u(w+w_2)}{r}, \Omega(w_2), \Omega(w)\right\} dF(w_2).$$

Define the operator $\mathbb{T} : C(\mathcal{W}) \to C(\mathcal{W})$ as

$$\mathbb{T}g(w) = \frac{u(w+b)}{\alpha+r} + \frac{\alpha}{\alpha+r} \int \max\left\{\frac{u(w+w_2)}{r}, g(w_2), g(w)\right\} dF(w_2),$$

where $C(\mathcal{W})$ is the space of bounded, continuous, and strictly increasing functions $g : \mathcal{W} \to \mathfrak{R}$ and $\mathcal{W} \equiv [0, \bar{w}]$. Since u is bounded, continuous, and strictly increasing and F is well-defined and continuous over the compact space $\mathcal{W}$, $\mathbb{T}$ maps the space of bounded, continuous and strictly increasing functions into itself. Moreover, since u is strictly increasing and $\frac{\alpha}{\alpha+r} \in (0,1)$ the operator $\mathbb{T}$ satisfies monotonicity and discounting, which are the sufficient conditions to apply Blackwell's theorem. Hence, $\mathbb{T}$ is a contraction mapping and the fixed point Ω is continuous and strictly increasing. ■

Proof. [Lemma 2] Recall that $\phi^-(w_1) = w_1$. Using the equation characterizing $\phi^+(w_1)$, equation (10), together with equations (6) and (4), we get

$$u\left(w_1 + \phi^+(w_1)\right) - u(w_1+b) = \alpha \int \max\left\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\right\} dF(w_2).$$

First of all, the continuity of u and Ω implies that ϕ^+ is continuous. Since the RHS of the above equation is bounded below by 0, $\phi^+(w_1) \geq b > 0$. So, $\lim_{w_1 \to 0} \phi^+(w_1) > \lim_{w_1 \to 0} \phi^-(w_1) = 0$. As $w_1 \to \bar{w}$, $\phi^+(w_1) < \phi^-(w_1) = \bar{w}$, otherwise the RHS of the above equation becomes zero, whereas the LHS does not. Given these facts, and using the intermediate value theorem,

ϕ^+ and ϕ^- should intersect at least at one point. We now prove part (i). Denoting $\hat{w}$ as the first of such intersection points, since $\lim_{w_1 \to 0} [\phi^+ (w_1) - \phi^- (w_1)] > 0$, for all $w_1 \leq \hat{w}$ we have $\phi (w_1) = \min \{\phi^+ (w_1), \phi^- (w_1)\} = \phi^- (w_1) = w_1$. We next prove part (ii), the existence of $\tilde{w}$. Above we showed that $\lim_{w_1 \to \bar{w}} \phi^+ (w_1) < \lim_{w_1 \to \bar{w}} \phi^- (w_1) = \bar{w}$, which implies $\lim_{w_1 \to \bar{w}} \phi (w_1) = \lim_{w_1 \to \bar{w}} \phi^+ (w_1)$. Recall that the quit/stay decision is also characterized by the ϕ^+ function and that $\lim_{w_2 \to \bar{w}} \phi^+ (w_2) < \bar{w}$. Thus, by continuity of ϕ^+, there exists a wage $\tilde{w} < \bar{w}$ such that the quit option is never exercised for any $w > \tilde{w}$, i.e., in that range $w_1 > \phi^+ (w_2)$. ■

Proof. [Proposition 1] We conjecture that, with risk-neutrality, we can write the value functions for the joint-search problem as:

$$
\begin{aligned}
T (w_1, w_2) &= W (w_1) + W (w_2) && \text{(A2)} \\
\Omega (w_1) &= W (w_1) + V && \text{(A3)} \\
U &= 2V. && \text{(A4)}
\end{aligned}
$$

Now, we verify our conjecture. $(A2)$ is straightforward. To verify $(A3)$, rewrite (6) using the conjecture for Ω:

$$
\begin{aligned}
r\Omega (w_1) &= w_1 + b + \alpha \int \max \{W (w_1) + W (w_2) - W (w_1) - V, W (w_1) + V - W (w_1) - V, 0\} dF (w_2) \\
&= w_1 + b + \alpha \int \max \{W (w_2) - V, 0\} dF (w_2) \\
&= rW (w_1) + rV.
\end{aligned}
$$

The second line uses the fact that the term $W(w_2) - W(w_1)$ is always dominated either by zero or by $W(w_2) - V$, the other two arguments of the max operator. Using $(A2)$ and $(A3)$ into equation (10) defining ϕ^+, one obtains $\phi^+ (w_1) = w^*$.

Similarly, substituting the conjecture for U into (5), we get

$$
\begin{aligned}
rU &= 2 \left(b + \alpha \int \max \{W (w) + V - 2V, 0\} dF (w) \right) \\
&= 2 \left(b + \alpha \int \max \{W (w) - V, 0\} dF (w) \right) \\
&= 2rV.
\end{aligned}
$$

We next show that $w^{**} = \hat{w} = \tilde{w} = w^*$, and hence $\phi = \phi^+$ for all $w \geq w^{**}$. Using $(A3)$ and $(A4)$ into $\Omega (w^{**}) = U$, the equation characterizing w^{**}, we obtain $W(w^{**}) = V$, which is the reservation wage equation for the single agent. Hence, $w^{**} = w^{*}$. The smallest double indifference wage $\hat{w}$ is characterized by the condition $\Omega(\hat{w}) = T(\hat{w}, \hat{w})$. Using $(A2)$ and $(A3)$, this equation becomes $V = W(\hat{w})$, which proves that $w^{*} = \hat{w}$. Since $\phi^{+}(w_1) = \hat{w}$, it must also be that $\hat{w} = \tilde{w}$ because there will be no quits beyond $\hat{w}$, which concludes the proof. ■

Proof. [Proposition 2] Since $r\Omega(\hat{w}) = u(2\hat{w})$ and $r\Omega(w^{**}) = rU$, using (5) we obtain

$$r \; [\Omega(\hat{w}) - \Omega(w^{**})] = u(2\hat{w}) - u(2b) - 2\alpha \int \max\{\Omega(w) - \; \Omega(w^{**}), 0\} \, dF(w) . \tag{A5}$$

At $w_1 = \hat{w}$, we can write equation (6) as

$$r\Omega(\hat{w}) = u(\hat{w} + b) + \alpha \int \max\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w}), 0\} \, dF(w) .$$

Multiplying by 2 and using the fact that $r\Omega(\hat{w}) = u(2\hat{w})$, we arrive at

$$u(2\hat{w}) = 2u(\hat{w} + b) - u(2\hat{w}) + 2\alpha \int \max\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w}), 0\} \, dF(w) .$$

Substituting this expression for $u(2\hat{w})$ into the RHS of equation $(A5)$ delivers

$$\begin{aligned} r \; [\Omega(\hat{w}) - \Omega(w^{**})] &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) \\ &+ 2\alpha \int \max\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w}), 0\} \, dF(w) \\ &- 2\alpha \int \max\{\Omega(w) - \; \Omega(w^{**}), 0\} \, dF(w) \end{aligned}$$

Now, by strict concavity of u, using Jensen's inequality we have $2u(\hat{w} + b) - u(2\hat{w}) - u(2b) > 0$. Suppose, ad absurdum, $w^{**} \geq \hat{w}$. Then, the RHS of the above equation is strictly positive, but the LHS is nonpositive, which is a contradiction. Therefore, $w^{**} < \hat{w}$. ■

Proof. [Proposition 3] By Lemma 2, we know that, beyond $\tilde{w}$, the quit option is never exercised and $\phi = \phi^{+}$. Using this result, substituting (6) into (10), and using $rT(w_1, w_2) = u(w_1 + w_2)$ we arrive at:

$$u(w_1 + \phi(w_1)) = u(w_1 + b) + \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2) .$$

Rearranging, we get

$$1 = \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) . \tag{A6}$$

We now study the shape of $\phi(w_1)$. Pratt (1964, Theorem 1) shows that if u belongs to the HARA family, for any $k > 0$ and m, n, p, q such that $p < q \leq m < n$, we have

$$f'(k) \begin{cases} > 0 & \text{if } u \text{ is DARA} \\ = 0 & \text{if } u \text{ is CARA} \\ < 0 & \text{if } u \text{ is IARA} \end{cases} ,$$

where

$$f(k) = \frac{u(n + k) - u(m + k)}{u(q + k) - u(p + k)} .$$

Setting $p = b$, $q = m = \phi(w_1)$, $n = w_2$, and $k = w_1$, it is straightforward to see that the expression inside the integral in equation ($A6$) is independent of w_1 in the CARA case, strictly increasing in w_1 in the DARA case, and strictly decreasing in w_1 in the IARA case, for any $w_2 > \phi(w_1)$. Moreover, since u is strictly increasing and the integral in ($A6$) is positive, the RHS of equation ($A6$) is strictly decreasing in $\phi(w_1)$. Therefore, for the equality in equation ($A6$) to hold, $\phi(w_1)$ must be independent of w_1 in the CARA case, strictly increasing in w_1 in the DARA case, and strictly decreasing in w_1 in the IARA case.

We now prove that $\tilde{w} = \hat{w}$ for CARA and IARA preferences. Starting with the conjecture that $\tilde{w} = \hat{w}$, the above results show that, for all $w_1 \geq \hat{w}$, indeed the reservation wage functions will be nonincreasing, and the quit option is never exercised. This verifies our conjecture, and hence $\tilde{w} = \hat{w}$. In the DARA case, since ϕ is strictly increasing beyond $\tilde{w}$, and $\tilde{w}$ is defined as the point where there is no quit anymore, we need to have $\tilde{w} > \hat{w}$.

Lastly, we establish the relation between w^* and $\hat{w}$. Consider the CARA case first. If u belongs to the CARA family, then $u(c_1 + c_2) = -u(c_1) u(c_2) / \rho$. Using this property, we can write equation ($A6$) as:

$$\begin{aligned} 1 &= \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_2) - u(\phi(w_1))}{u(\phi(w_1)) - u(b)} \right] dF(w_2) , \\ u(\phi(w_1)) &= u(b) + \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_2) - u(\phi(w_1))] \, dF(w_2) , \end{aligned}$$

which is exactly equation (3) characterizing the reservation wage of the single agent. Since $\phi(w_1)$ is constant, $\hat{w} = w^*$.

We now move to the DARA case. Using $r\Omega(\hat{w}) = u(2\hat{w})$, we can rewrite equation (6) evaluated at $\hat{w}$ as

$$\begin{aligned} u(2\hat{w}) - u(\hat{w}+b) &= \frac{\alpha}{r}\int \max\left\{rT(\hat{w},w) - u(2\hat{w}), r\Omega(w) - u(2\hat{w}), 0\right\} dF(w) \\ &\geq \frac{\alpha}{r}\int \max\left\{rT(\hat{w},w) - u(2\hat{w}), 0\right\} dF(w) \\ &= \frac{\alpha}{r}\int_{\hat{w}} \left[u(\hat{w}+w) - u(\hat{w}+\hat{w})\right] dF(w). \end{aligned}$$

Rearrange the above equation to get

$$\begin{aligned} 1 &\geq \frac{\alpha}{r}\int_{\hat{w}} \left[\frac{u(\hat{w}+w) - u(\hat{w}+\hat{w})}{u(\hat{w}+\hat{w}) - u(\hat{w}+b)}\right] dF(w) \\ &> \frac{\alpha}{r}\int_{\hat{w}} \left[\frac{u(w) - u(\hat{w})}{u(\hat{w}) - u(b)}\right] dF(w), \end{aligned}$$

where the second inequality uses the property of DARA preferences. We know from equation (3) that

$$1 = \frac{\alpha}{r}\int_{w^*} \left[\frac{u(w) - u(w^*)}{u(w^*) - u(b)}\right] dF(w),$$

and since its RHS is a strictly decreasing function of w^*, it is easy to see that $w^* < \hat{w}$.

Finally, in the IARA case, we can write equation (6) evaluated at $\hat{w}$ as

$$\begin{aligned} u(2\hat{w}) - u(\hat{w}+b) &= \frac{\alpha}{r}\int_{\hat{w}} \left[rT(\hat{w},w) - u(2\hat{w})\right] dF(w). \\ &= \frac{\alpha}{r}\int_{\hat{w}} \left[u(\hat{w}+w) - u(\hat{w}+\hat{w})\right] dF(w), \end{aligned}$$

because at wage $\hat{w}$ the employed spouse does not quit his job whenever the unemployed spouse accepts her job offer, by virtue of the fact that ϕ is strictly decreasing, as shown above. Rearranging the equation above and comparing it to the single agent reservation wage equation yields

$$\begin{aligned} \frac{\alpha}{r}\int_{w^*} \left[\frac{u(w) - u(w^*)}{u(w^*) - u(b)}\right] dF(w) = 1 &= \frac{\alpha}{r}\int_{\hat{w}} \left[\frac{u(\hat{w}+w) - u(\hat{w}+\hat{w})}{u(\hat{w}+\hat{w}) - u(\hat{w}+b)}\right] dF(w) \\ &< \frac{\alpha}{r}\int_{\hat{w}} \left[\frac{u(w) - u(\hat{w})}{u(\hat{w}) - u(b)}\right] dF(w), \end{aligned}$$

where the inequality in the second line follows from the IARA property. Therefore, $\hat{w} < w^*$.

This concludes the proof. ■

Proof. [Proposition 4] We begin with part (i). The value functions (4) and (6) modified to allow for exogenous separations are

$$rT(w_1, w_2) = u(w_1 + w_2) + \delta[\Omega(w_1) - T(w_1, w_2)] + \delta[\Omega(w_2) - T(w_1, w_2)] \tag{A7}$$

$$r\Omega(w_1) = u(w_1 + b) + \delta[U - \Omega(w_1)] \tag{A8}$$

$$+ \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} \, dF(w_2). \tag{A9}$$

First of all, notice that using the same arguments as in the proof of Lemma 1, we can show T and Ω are continuous and strictly increasing functions. From the definition of ϕ^+ for the worker-searcher couple, $T(w_1, \phi^+(w_1)) = \Omega(w_1)$, we have:

$$\begin{aligned} rT\left(w_1, \phi^+(w_1)\right) &= r\Omega(w_1) \\ u\left(w_1 + \phi^+(w_1)\right) - u(w_1 + b) &= \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} \, dF(w_2) \quad \text{(A10)} \\ &\quad + \delta\left[\Omega(w^{**}) - \Omega\left(\phi^+(w_1)\right)\right] \end{aligned}$$

Since Ω is strictly increasing, using the same arguments as in the proof of Lemma 2, we can again show that $\lim_{w_1 \to \bar{w}} \phi^+(w_1) < \bar{w}$, which establishes the result.

Next, we prove part (ii). By the definition of $\hat{w}$, for all $w_1 \leq \hat{w}$, we have $\phi(w_1) = \phi^-(w_1) = w_1$. Moreover, by the definition of $\tilde{w}$, there is no quit beyond $\tilde{w}$ and $\phi = \phi^+$. Then, the second argument in the max operator in equation $(A8)$ becomes irrelevant, and we are left with the following equation:

$$\begin{aligned} u(w_1 + \phi(w_1) &= u(w_1 + b) + \alpha \int_{\phi(w_1)} [T(w_1, w_2) - T(w_1, \phi(w_1))] \, dF(w_2) - \delta[\Omega(\phi(w_1)) - U] \\ &= u(w_1 + b) + h(\phi(w_1)) \qquad \text{(A11)} \\ &\quad + \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2), \end{aligned}$$

where

$$h(x) = \frac{\alpha\delta}{r + 2\delta} \int_x [\Omega(w_2) - \Omega(x)] \, dF(w_2) - \delta[\Omega(x) - U]$$

with h decreasing in x. Rearrange equation $(A11)$ as

$$1 = \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}\right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}. \tag{A12}$$

If we assume $\phi(w_1)$ is a nonincreasing function of w_1 beyond $\tilde{w}$, then, for any $w_1' > w_1 \geq \tilde{w}$, we have

$$\begin{aligned} 0 &\leq \frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} \leq \frac{u(w_1'+w_2) - u(w_1'+\phi(w_1))}{u(w_1'+\phi(w_1)) - u(w_1'+b)} \\ &\leq \frac{u(w_1'+w_2) - u(w_1'+\phi(w_1'))}{u(w_1'+\phi(w_1')) - u(w_1'+b)}, \end{aligned}$$

where the second weak inequality stems from the fact that u is CARA or DARA, and the third from the fact that ϕ is weakly decreasing. Overall, the above condition implies the first term in equation $(A12)$ is an increasing function of w_1.

Since for $w_1 \geq \tilde{w}$, h is decreasing in x, and $\phi(w_1') \leq \phi(w_1)$, we have

$$\frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} < \frac{h(\phi(w_1'))}{u(w_1'+\phi(w_1')) - u(w_1'+b)},$$

because the right hand side has a weakly greater numerator and a strictly smaller denominator than the left-hand side. We reach the following contradiction for $w_1' > w_1 \geq \tilde{w}$:

$$\begin{aligned} 1 &= \frac{\alpha}{r+2\delta}\int_{\phi(w_1)}\left[\frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right]dF(w_2) + \frac{h(\phi(w_1))}{u(w_1+\phi^+(w_1)) - u(w_1+b)} \\ &< \frac{\alpha}{r+2\delta}\int_{\phi(w_1')}\left[\frac{u(w_1'+w_2) - u(w_1'+\phi(w_1'))}{u(w_1'+\phi(w_1')) - u(w_1'+b)}\right]dF(w_2) + \frac{h(\phi(w_1'))}{u(w_1'+\phi(w_1')) - u(w_1'+b)} \\ &= 1, \end{aligned}$$

where the last equality follows from the fact that the RHS in the second line is like the RHS in the first line evaluated at w_1' instead of w_1. We conclude that $\phi(w_1)$ is strictly increasing in w_1.

Lastly, we prove part (iii). Since ϕ is strictly increasing beyond $\tilde{w}$, the definitions of $\tilde{w}$ and $\hat{w}$ make it clear that $\tilde{w} > \hat{w}$. To prove $w^{**} < \hat{w}$, evaluating equation (A7) at $(\hat{w}, \hat{w})$ and using the fact $T(\hat{w}, \hat{w}) = \Omega(\hat{w})$, we get

$$r\Omega(\hat{w}) = rT(\hat{w}, \hat{w}) = u(2\hat{w}).$$

The rest of the proof follows closely the proof of Proposition 2 using equation $(A10)$. ■

Proof. [Proposition 5] We first prove parts (i) and (ii), which establish the behavior of the reservation wage functions. It is easy to see that T, S, and Ω are continuous and strictly increasing functions using arguments similar to those in the proof of Lemma 1. The reservation function for an outside offer satisfies $\phi_o(w_1) = \min\{\phi_o^+(w_1), w_1\}$, where $\phi_o^+(w_1)$ solves $S(w_1, \phi_o^+(w_1)) = \Omega(w_1)$. Similarly, we have $\phi_i(w_1) = \min\{\phi_i^+(w_1), w_1\}$, where $\phi_i^+(w_1)$ solves $T\left(w_1, \phi_i^+(w_1)\right) = \Omega(w_1)$. As before, we begin by conjecturing that the quit option is never exercised beyond a certain wage threshold, $\hat{w}_S$ for outside offers and inside offers. Then for $w_1 \geq \hat{w}_S$, we have $\phi_o(w_1) = \phi_o^+(w_1)$ and $\phi_i(w_1) = \phi_i^+(w_1)$. Using the definition of $\phi_o^+(w_1)$, we can characterize $\phi_o(w_1)$ as:

$$\begin{aligned}
\phi_o(w_1) &= b + \kappa + \alpha_i \int_{\phi_i(w_1)} [T(w_1, w_2) - \Omega(w_1)]\, dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)]\, dF(w_2) \\
&= b + \kappa + \alpha_i \int_{\phi_i(w_1)} T_2(w_1, w_2)(1 - F(w_2))\, dw_2 + \alpha_o \int_{\phi_o(w_1)} S_2(w_1, w_2)(1 - F(w_2))\, dw_2 \\
&= b + \kappa + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F(w_2)]\, dw_2 + \frac{\alpha_o}{r} \int_{\phi_o(w_1)} [1 - F(w_2)]\, dw_2, \qquad \text{(A13)}
\end{aligned}$$

where the second line is obtained through integration by parts and the third line uses the risk neutrality assumption, which assures $T_2(w_1, w_2) = S_2(w_1, w_2) = \frac{1}{r}$.

We now turn to inside offers. We continue to analyze the region of wage offers $w_1 \geq \hat{w}_S$, where we know the employed worker does not quit upon receiving outside offers. Since, in this range, $\phi_i(w_1) = \phi_i^+(w_1)$, using the definition of $\phi_i^+(w_1)$ we have:

$$\begin{aligned}
\phi_i(w_1) &= b + \alpha_i \int_{\phi_i(w_1)} [T(w_1, w_2) - \Omega(w_1)]\, dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)]\, dF(w_2) \\
&= b + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F(w_2)]\, dw_2 + \frac{\alpha_o}{r} \int_{\phi_o(w_1)} [1 - F(w_2)]\, dw_2, \qquad \text{(A14)}
\end{aligned}$$

where the second line is derived exactly as for the outside offer case.

Combining equations ($A13$) and ($A14$), we can verify that $\phi_o(w_1)$ and $\phi_i(w_1)$ are independent of w_1, and $\phi_o(w_1) = \hat{w}_S = \phi_i^+(w_1) + \kappa$ for $w_1 \geq \hat{w}_S$. This yields $\phi_o(w_1) = \hat{w}_S$ for $w_1 \geq \hat{w}_S$, and confirms the conjecture that the employed spouse with wage $w_1 > \hat{w}_S$ never quits upon the unemployed accepting an outside offer, since the quit/stay decision for the employed is just the mirror image of the accept/reject decision for the unemployed according to 45^0 line due to the symmetry of agents.

Defining $\hat{w}_T = \hat{w}_S - \kappa$, in the region, $w_1 \geq \hat{w}_S$, for inside offers we get $\phi_i(w_1) = \hat{w}_T < w_1$, given the conjecture that the employed does not quit in this region for inside offers to the unemployed. We will verify this conjecture together with the characterization of ϕ_i.

Let us extend our analysis of inside offers to the region where $w_1 < \hat{w}_S$. Here, we conjecture that for inside offers the employed never quits, i.e., $\phi_i = \phi_i^+$. Notice that given this conjecture, for outside offers, the employed with wage $w_1 < \hat{w}_S$ always quits whenever the unemployed accepts a job offer. If she did not quit, the equations characterizing ϕ_o and ϕ_i would become ($A13$) and ($A14$), respectively. But then, $\phi_o(w_1) = \hat{w}_S > w_1$ since $\phi_o^-(w_1) = w_1 < \hat{w}_S = \phi_o^+(w_1)$, which contradicts the conjecture that employed pouse does not quit if his wage is equal to w_1. This result implies that, for $w_1 < \hat{w}_S$, we need to have $\phi_o(w_1) = w_1$ for outside offers:

$$\phi_o(w_1) = \begin{cases} \hat{w}_S & \text{for } w_1 \geq \hat{w}_S \\ w_1 & \text{for } w_1 < \hat{w}_S \end{cases}$$

Then, using the definition of ϕ_i^+, we get:

$$\phi_i^+(w_1) = b + \frac{\alpha_i}{r}\int_{\phi_i^+(w_1)} [1 - F(w)]\,dw + \alpha_o \int_{w_1} [\Omega(w_2) - \Omega(w_1)]\,dF(w_2).$$

Since the last term is strictly decreasing in w_1, $\phi_i^+(w_1)$ is strictly decreasing in w_1 over this region. We also know that $\phi_i^+(\hat{w}_S) = \hat{w}_T < \hat{w}_S$. So, there exists $\hat{w} \in (\hat{w}_T, \hat{w}_S)$, the double indifference point, such that $\phi_i^+(w_1) < w_1$ for $w_1 > \hat{w}$, and $\phi_i^+(w_1) \geq w_1$ for $w_1 \leq \hat{w}$.

$$\phi_i(w_1) = \begin{cases} \phi_i^+(w_1) < w_1 & \text{for } w_1 > \hat{w} \\ w_1 & \text{for } w_1 \leq \hat{w} \end{cases}$$

Again, using the symmetry argument, we can verify that indeed the employed with wage $w_1 \geq \hat{w}$ never quits upon the unemployed receiving an inside offer. This completes the proof of parts (i) and (ii).

We next prove part (iii) of the proposition: $w^{**} \in (\hat{w}_T, \hat{w})$ and $w^* \in (\hat{w}, \hat{w}_S)$, so $w^{**} < w^*$. It is also useful to recall that $\hat{w}_T < \hat{w} < \hat{w}_S$.

Step 1: We first show $w^{**} > \hat{w}_T$. Equation (19) evaluated at the point $w_1 = \hat{w}_T$ becomes

$$r\Omega(\hat{w}_T) = \hat{w}_T + b + (\alpha_i + \alpha_o)\,g(\hat{w}_T) \tag{A15}$$

where

$$g(w) = \int_w [\Omega(w') - \Omega(w)]\,dF(w')$$

The reservation wage of the dual-searcher couple w^{**} is characterized by the equation

$$r\Omega(w^{**}) = 2b + 2(\alpha_i + \alpha_o)\,g(w^{**}). \tag{A16}$$

Now subtract equation $(A15)$ multiplied by 2 from equation $(A16)$ and get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}_T\right)\right]=r\Omega\left(\hat{w}_T\right)-2\hat{w}_T+2\left(\alpha_i+\alpha_o\right)\left(g\left(w^{**}\right)-g\left(\hat{w}_T\right)\right).$$

Suppose $w^{**}\leq\hat{w}_T$, then the LHS of the above equation is nonpositive. The third term of the RHS is nonnegative since $g\left(w\right)$ is strictly decreasing in w due to $\Omega\left(w\right)$ being strictly increasing in w. The term $r\Omega\left(\hat{w}_T\right)-2\hat{w}_T$ is positive because for $w_1=\hat{w}_T$, $\Omega\left(\hat{w}_T\right)>T\left(\hat{w}_T,\hat{w}_T\right)=\frac{2\hat{w}_T}{r}$, the employed worker would prefer to quit his job rather than remain employed (more precisely, he strictly prefers it for an outside offer, but he is indifferent for an inside offer). Therefore the RHS is positive, which is a contradiction. So $w^{**}>\hat{w}_T$.

Step 2: Similarly, consider equation (19) evaluated at $w_1=\hat{w}$. Note that at $w_1=\hat{w}$, for inside offers the employed spouse never exercises the quit option, whereas for outside offers, she does. So, equation (19) evaluated at $w_1=\hat{w}$ becomes

$$r\Omega\left(\hat{w}\right)=\hat{w}+b+\alpha_i\int_{\hat{w}}\left[T\left(\hat{w},w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right)+\alpha_o g\left(\hat{w}\right).$$

Also note that since $\hat{w}$ is the double indifference point for inside offers, $r\Omega\left(\hat{w}\right)=2\hat{w}$. Again, subtract this last equation multiplied by 2 from equation $(A16)$ and use the fact that $r\Omega\left(\hat{w}\right)=2\hat{w}$ to get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}\right)\right]=2\alpha_i\left[g\left(w^{**}\right)-\int_{\hat{w}}\left[T\left(\hat{w},w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right)\right]+2\alpha_o\left(g\left(w^{**}\right)-g\left(\hat{w}\right)\right).$$

Now, suppose $w^{**}\geq\hat{w}$. Then the LHS becomes nonnegative. The last term in the RHS is nonpositive. Moreover $\int_{\hat{w}}\left[T\left(\hat{w},w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right)\geq\int_{w^{**}}\left[T\left(w^{**},w\right)-\Omega\left(w^{**}\right)\right]dF\left(w\right)>\int_{w^{**}}\left[\Omega\left(w\right)-\Omega\left(w^{**}\right)\right]dF\left(w\right)=g(w^{**})$, since $T\left(w^{**},w\right)>\Omega\left(w\right)$ for $w>w^{**}\geq\hat{w}$. Therefore, the first term on the RHS must be negative, which delivers a contradiction and leads to $w^{**}<\hat{w}$. Steps 1 and 2 establish that $w^{**}\in\left(\hat{w}_T,\hat{w}\right)$.

Step 3: We next prove $w^{*}>\hat{w}$. First of all, notice that the equation characterizing the reservation wage of the single becomes

$$w^{*}=b+\frac{\alpha_i+\alpha_o}{r}\int_{w^{*}}\left[1-F\left(w\right)\right]dw. \tag{A17}$$

Combining equation (19) evaluated at $\hat{w}$ with the fact that $r\Omega\left(\hat{w}\right)=2\hat{w}$, we have

$$\hat{w}=b+\frac{\alpha_i}{r}\int_{\hat{w}}\left[1-F\left(w\right)\right]dw+\alpha_o\int_{\hat{w}}\left[\Omega\left(w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right).$$

Subtracting this equation from equation $(A17)$, we get

$$w^* - \hat{w} = \frac{\alpha_i}{r}\int_{w^*}^{\hat{w}} [1 - F(w)]\, dw + \frac{\alpha_o}{r}\left[\int_{w^*} [w - w^*]\, dF(w) - \int_{\hat{w}} [r\Omega(w) - r\Omega(\hat{w})]\, dF(w)\right].$$

Using the fact that $r\Omega(w) = rT(w + \phi_i^+(w)) = w + \phi_i^+(w)$ for $w \geq \hat{w}$, we get

$$w^* - \hat{w} = \frac{\alpha_i + \alpha_o}{r}\int_{w^*}^{\hat{w}} [1 - F(w)]\, dw - \frac{\alpha_o}{r}\int_{\hat{w}} \left[\phi_i^+(w) - \phi_i^+(\hat{w})\right] dF(w)\,.$$

Suppose $w^* \leq \hat{w}$, then the LHS becomes nonpositive, and the first term on the RHS is nonnegative. Since $\phi_i^+(w)$ is strictly decreasing in the range $w \in (\hat{w}, \hat{w}_S)$ and constant for $w \geq \hat{w}_S$, the last term on the RHS is positive, a contradiction. Thus, $w^* > \hat{w}$.

Step 4: Finally we show that $w^* < \hat{w}_S$. Rewrite the equation for $\hat{w}_S$ as

$$\hat{w}_S = b + \kappa + \frac{\alpha_i}{r}\int_{\hat{w}_S - \kappa} (1 - F(w))\, dw + \frac{\alpha_o}{r}\int_{\hat{w}_S} (1 - F(w))\, dw.$$

Subtracting equation $(A17)$ from the equation defining $\hat{w}_S$, we get

$$\hat{w}_S - w^* = \kappa + \frac{\alpha_i}{r}\int_{\hat{w}_S - \kappa}^{w^*} [1 - F(w)]\, dw + \frac{\alpha_o}{r}\int_{\hat{w}_S}^{w^*} [1 - F(w)]\, dw.$$

Suppose $w^* \geq \hat{w}_S$, then the LHS is non-positive. However, since $\kappa > 0$, the RHS is strictly positive. Thus, $w^* < \hat{w}_S$. Therefore, $w^* \in (\hat{w}, \hat{w}_S)$, and the proof is complete. ■

B Additional Results

We formally state and prove the additional results discussed in Section 3.5

B.1 Consumption as a Private Good

Proposition 6 *With CARA preferences, when consumption is a private good and spouses have equal weight in household utility, $w^* < \hat{w}$.*

Proof. With CARA preferences, since the employed never quits beyond $\hat{w}$, we can write equation (6) evaluated at $w_1 = \hat{w}$ as

$$\begin{aligned}
1 &= \frac{\alpha}{r}\int_{\hat{w}} \frac{u\left(\frac{\hat{w}}{2}+\frac{w}{2}\right)-u\left(\frac{\hat{w}}{2}+\frac{\hat{w}}{2}\right)}{u\left(\frac{\hat{w}}{2}+\frac{\hat{w}}{2}\right)-u\left(\frac{\hat{w}}{2}+\frac{b}{2}\right)}dF(w) \\
&= \frac{\alpha}{r}\int_{\hat{w}} \frac{u\left(\frac{w}{2}\right)-u\left(\frac{\hat{w}}{2}\right)}{u\left(\frac{\hat{w}}{2}\right)-u\left(\frac{b}{2}\right)}dF(w) \\
&> \frac{\alpha}{r}\int_{\hat{w}} \frac{\left[u\left(\frac{w}{2}\right)\right]^2-\left[u\left(\frac{\hat{w}}{2}\right)\right]^2}{\left[u\left(\frac{\hat{w}}{2}\right)\right]^2-\left[u\left(\frac{b}{2}\right)\right]^2}dF(w) \\
&= \frac{\alpha}{r}\int_{\hat{w}} \frac{u(w)-u(\hat{w})}{u(\hat{w})-u(b)}dF(w),
\end{aligned}$$

where the second and fourth lines are due to the CARA assumption, and the third line uses the fact that u is CARA and strictly increasing and concave which implies $|u(y)| > |u(x)|$ if $y < x$. Recall that the equation characterizing the reservation wage for the single agent, w^*, can be written as

$$1 = \frac{\alpha}{r}\int_{w^*} \frac{u(w)-u(w^*)}{u(w^*)-u(b)}dF(w).$$

Then, it is immediate to see $\hat{w} > w^*$. ■

B.2 CARA Utility with Borrowing

Before analyzing the joint-search problem, it is useful to recall here the solution to the single-agent problem with CARA preferences in the presence of borrowing and saving.

Single-agent search problem. Let a denote the asset position of the individual. Assets evolve according to the law of motion,

$$\frac{da}{dt} = ra + y - c, \tag{B1}$$

where r is the risk-free interest rate, y is income (equal to w during employment and b during unemployment), and c is consumption. The value functions for the employed and unemployed single agent are, respectively:

$$rW(w,a) = \max_c \left\{u(c) + W_a(w,a)(ra+w-c)\right\}, \tag{B2}$$

$$rV(a) = \max_c \left\{u(c) + V_a(a)(ra+b-c)\right\} + \alpha\int \max\left\{W(w,a)-V(a),0\right\}dF(w), \tag{B3}$$

where the subscript denotes the partial derivative. These equations reflect the nonstationarity due to the change in assets over time. For example, the second term on the RHS of (B2) is $(dW/dt) = (dW/da) \cdot (da/dt)$. And similarly for the second term on the RHS of (B3).

We begin by conjecturing that $rW(w,a) = u(ra+w)$. If this is the case, then the first-order condition (FOC) determining optimal consumption for the agent gives $u'(c) = u'(ra+w)$, which confirms the conjecture and establishes that the employed individual consumes his current wage plus the interest income on the risk-free asset. Let us now guess that $rV(a) = u(ra+w^*)$. Once gain, it is easy to verify this guess through the FOC of the unemployed agent. Substituting this solution back into equation (B3) and using the CARA assumption yields

$$w^* = b + \frac{\alpha}{\rho r} \int_{w^*} [u(w-w^*) - 1] \, dF(w), \tag{B4}$$

which shows that w^* is the reservation wage and is independent of wealth. Therefore, the unemployed worker consumes the reservation wage plus the interest income on his wealth. This result highlights an important point: the asset position of an unemployed worker deteriorates and, in the presence of a debt constraint, she may hit it. As in the rest of the papers that use this setup, we abstract from this possibility. The implicit assumption is that borrowing constraints are "loose," and by this we mean they do not bind along the solution for the unemployed agent.

Joint-search problem. When the couple searches jointly for jobs, the asset position of the couple still evolves based on (B1), but now $y = 2b$ for the dual-searcher couple, $b + w_1$ for the worker-searcher couple, and $w_1 + w_2$ for the employed couple. The value functions become

$$rT(w_1, w_2, a) = \max_c \{u(c) + T_a(w_1, w_2, a)(ra + w_1 + w_2 - c)\}, \tag{B5}$$

$$rU(a) = \max_c \{u(c) + U_a(a)(ra + 2b - c)\} + \alpha \int \max\{\Omega(w,a) - U(a), 0\} \, dF(w), \tag{B6}$$

$$\begin{aligned} r\Omega(w_1, a) &= \max_c \{u(c) + \Omega_a(w_1, a)(ra + w_1 + b - c)\} \\ &\quad + \alpha \int \max\{T(w_1, w_2, a) - \Omega(w_1, a), \Omega(w_2, a) - \Omega(w_1, a), 0\} \, dF(w_2). \end{aligned} \tag{B7}$$

Solving this problem requires characterizing the optimal consumption policy for the dual-searcher couple $c_u(a)$, for the worker-searcher couple $c_{eu}(w_1, a)$, and for the dual-worker couple $c_e(w_1, w_2, a)$, as well as the reservation wage functions, now potentially a function of wealth too, which must satisfy, as usual: $\Omega(w^{**}(a), a) = U(a)$, $T(w_1, \phi^+(w_1, a), a) = \Omega(w_1, a)$,

$\Omega\left(\phi^{-}\left(w_1,a\right),a\right)=\Omega\left(w_1,a\right)$ and $\phi(w_1,a)=min\{\phi^{+}(w_1,a),\phi^{-}(w_1,a)\}$. The following proposition characterizes the solution to this problem.

Proposition 7 ***[CARA Utility with Borrowing-Saving]*** *With CARA preferences, access to risk-free borrowing and lending, and "loose" debt constraints, the search behavior of a couple can be characterized as follows:*

(i) The optimal consumption policies are: $c_u\left(a\right)=ra+2w^{**}$, $c_{eu}\left(w_1,a\right)=ra+w^{**}+w_1$, *and* $c_e\left(w_1,w_2,a\right)=ra+w_1+w_2$.

(ii) The reservation function ϕ *of the worker-searcher couple is independent of* $\left(w_1,a\right)$ *and equals* w^{**}*, so there is no breadwinner cycle.*

(iii) The reservation wage w^{**} *of the dual-searcher couple equals* w^{*}*, the reservation wage of the single-agent problem.*

Proof. We conjecture that $rT\left(w_1,w_2,a\right)=u\left(ra+w_1+w_2\right)$. Then the RHS of equation (B5) becomes

$$\max_{c}\left\{u(c)+u'\left(ra+w_1+w_2\right)\left(ra+w_1+w_2-c\right)\right\}.$$

The FOC implies $u'\left(c\right)=u'\left(ra+w_1+w_2\right)$, so $c_e\left(a,w_1,w_2\right)=ra+w_1+w_2$. If we plug this optimal consumption function back into equation $(B5)$, we arrive at $rT\left(w_1,w_2,a\right)=\left(ra+w_1+w_2\right)$, which confirms the conjecture.

Similarly, let us guess that $r\Omega\left(w_1,a\right)=u\left(ra+w_1+\phi\left(w_1,a\right)\right)$. Again, plugging this guess into the RHS of equation $(B7)$, the FOC implies $c_{eu}\left(w_1,a\right)=ra+w_1+\phi\left(w_1,a\right)$. Substituting this function back into $(B7)$ gives

$$\begin{aligned}r\Omega\left(w_1,a\right)=&\,u(ra+w_1+\phi\left(w_1,a\right))+u'\left(ra+w_1+\phi\left(w_1,a\right)\right)\left(b-\phi\left(w_1,a\right)\right)\\&+\frac{\alpha}{r}\int\max\left\{u\left(ra+w_1+w_2\right)-u\left(ra+w_1+\phi\left(w_1,a\right)\right),\right.\\&\qquad\left.u\left(ra+w_2+\phi\left(w_1,a\right)\right)-u\left(ra+w_1+\phi\left(w_1,a\right)\right),0\right\}dF\left(w_2\right).\end{aligned}$$

Using the CARA property of u, we can simplify the RHS and rewrite the above equation as

$$\begin{aligned}r\Omega\left(w_1,a\right)\;=\;&u(ra+w_1+\phi\left(w_1,a\right))\left[1-\rho\left(b-\phi\left(w_1,a\right)\right)\right.\\&\left.-\frac{\alpha}{r}\int\max\left\{u\left(w_2-\phi\left(w_1,a\right)\right)-1,u\left(w_2-w_1\right)-1,0\right\}dF\left(w_2\right)\right].\end{aligned}$$

Now, using the definition of ϕ^+ and the expression for $rT(w_1, \phi^+(w_1,a), a)$ in the above equation, we have

$$\phi^+(w_1,a) = b + \frac{\alpha}{\rho r}\int \left[u\left(\max\left\{w_2 - \phi(w_1,a), w_2 - w_1, 0\right\}\right) - 1\right] dF(w_2).$$

First, notice that $\phi^-(w_1,a) = w_1$ for all a. Next, as in the CARA case without saving, conjecture that there is a value $w_1 = \hat{w}$ such that beyond that value the quitting option is never exercised which happens if and only if $\phi^+(w_1,a) < \phi^-(w_1,a)$, i.e. $\phi(w_1,a) = \phi^+(w_1,a)$. Then, in this range we can ignore the second argument in the max operator and rewrite

$$\phi(w_1,a) = \phi^+(w_1,a) = b + \frac{\alpha}{\rho r}\int_{\phi^+(w_1,a)} \left[u\left(w_2 - \phi^+(w_1,a)\right) - 1\right] dF(w_2), \tag{B8}$$

which implies that ϕ is a constant function, independent of (w_1,a). Moreover, comparing $(B8)$ to the analogous equation for the single-agent problem $(B4)$ yields that $\phi(w_1,a) = \hat{w} = w^*$. This also confirms our conjecture that there is no quit beyond $w_1 = \hat{w}$.

Finally, let us turn to U and conjecture that $rU(a) = u(ra + 2w^{**})$. Substituting this guess into equation $(B6)$ and taking the FOC leads to the optimal policy function $c_u(a) = ra + 2w^{**}$, which confirms the guess. Then, using the CARA assumption, equation $(B6)$ becomes

$$\begin{aligned} rU(a) &= u(ra+2w^{**}) - \rho u(ra+2w^{**})(2b - 2w^{**}) - \frac{2\alpha}{r}u(ra+2w^{**})\int_{w^{**}} \left[u(w - w^{**}) - 1\right] dF(w) \\ &= u(ra+2w^{**})\left[1 - \rho(2b - 2w^{**}) - \frac{2a}{r}\int_{w^{**}} \left[u(w - w^{**}) - 1\right] dF(w)\right] \end{aligned}$$

and using $rU(a) = u(ra + 2w^{**})$ we arrive at

$$w^{**} = b + \frac{a}{\rho r}\int_{w^{**}} \left[u(w - w^{**}) - 1\right] dF(w),$$

which, once again, compared to $(B4)$ implies that $w^{**} = w^*$. This concludes the proof. ■

The main message of this proposition could perhaps be anticipated by the fact that borrowing effectively substitutes for the consumption smoothing provided within the household, making the latter redundant. Each spouse can implement search strategies that are independent from the other spouse's actions and, as a result, each acts as in the single-agent model. Of course, to the extent that borrowing constraints bind or preferences deviate from CARA, the equivalence result no longer applies.

B.3 On-the-Job Search

Suppose that agents can search both off and on the job. During unemployment, they draw a new wage from $F(w)$ at rate α_u, whereas during employment they sample new job offers from the same distribution F at rate α_e. What we develop below is, essentially, a version of the Burdett (1978) wage ladder model with couples. The flow value functions in this case are

$$rU = u(2b) + 2\alpha_u \int \max\{\Omega(w) - U, 0\}\, dF(w), \tag{B9}$$

$$\begin{aligned} r\Omega(w_1) &= u(w_1 + b) + \alpha_u \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2) \\ &\quad + \alpha_e \int \max\{\Omega(w_1') - \Omega(w_1), 0\}\, dF(w_1'), \end{aligned} \tag{B10}$$

$$\begin{aligned} rT(w_1, w_2) &= u(w_1 + w_2) + \alpha_e \int \max\{T(w_1', w_2) - T(w_1, w_2), 0\}\, dF(w_1') \\ &\quad + \alpha_e \int \max\{T(w_1, w_2) - T(w_1, w_2), 0\}\, dF(w_2). \end{aligned} \tag{B11}$$

We continue to denote the reservation wage of the dual-searcher couple as w^{**} and the reservation wage of the unemployed spouse in the worker-searcher couple as $\phi(w_1)$. We now have a new reservation function, that of the employed spouse (in the dual-worker couple and in the worker-searcher couple) which we denote by $\eta(w_i)$. It is intuitive (and can be proved easily) that under risk neutrality the joint-search problem coincides with the problem of the single agent regardless of offer arrival rates. Below, we prove another equivalence result that holds for any concave utility function but for the special case of symmetric offer arrival rates $\alpha_u = \alpha_e$, i.e., when search is equally effective on and off the job.

Proposition 8 *[On-the-job Search with Symmetric Arrival Rates]* *If $\alpha_u = \alpha_e$, the joint-search problem yields the same solution as the single-agent search problem, even with concave preferences. Specifically, $w^{**} = w^* = b$, $\phi(w_1) = w^{**}$ and $\eta(w_i) = w_i$ for $i = 1, 2$.*

Proof. Let us conjecture that $\phi(w_1) = w^{**}$ for any value of w_1, i.e., $T(w^{**}, w_1) = \Omega(w_1)$. This implies that the quit option is never exercised, since any observed w_1 will be greater than or equal to w^{**}. So, one can disregard the second argument in the max operator in $(B10)$. Evaluating $(B10)$ at w^{**} yields

$$r\Omega(w^{**}) = u(w^{**} + b) + 2\alpha_u \int \max\{\Omega(w) - \Omega(w^{**}), 0\}\, dF(w),$$

where we have used the fact that $\alpha_e = \alpha_u$ and the conjecture. Since $\Omega(w^{**}) = U$, comparing the above equation to $(B9)$ yields that $w^{**} = b$. We now verify our conjecture. From $(B11)$ evaluated at $w_2 = w^{**}$, we obtain

$$\begin{aligned} rT\left(w_1, w^{**}\right) &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{T\left(w_1', w^{**}\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_e \int \max\left\{T\left(w_1, w_2\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_2\right) \\ &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{\Omega\left(w_1'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_u \int \max\left\{T\left(w_1, w_2\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2\right) \\ &= \Omega\left(w_1\right), \end{aligned}$$

which confirms our conjecture, since $T\left(w^{**}, w_2\right) = \Omega\left(w_2\right)$ implies that $\phi\left(w_2\right) = w^{**}$. Finally, from equation $(B11)$, it is immediate that $\eta\left(w_i\right) = w_i$, which completes the proof. ■

C SIPP Sample Selection

Table C1: Summary Statistics of SIPP Sample

	Singles		Couples	
	Male	Female	Male	Female
Age	27.26	27.48	32.00	30.45
	(5.53)	(5.44)	(4.94)	(5.09)
Years of Education	12.54	12.83	12.95	13.29
	(2.32)	(2.30)	(2.20)	(2.21)
Nonwhite	0.22	0.34	0.11	0.11
	(0.41)	(0.47)	(0.33)	(0.33)
Number of Children	0.24	0.59	1.50	
	(0.59)	(0.846)	(0.984)	
Hourly Wage ($)	9.19	8.55	14.51	11.74
	(4.80)	(4.37)	(26.55)	(14.70)
Wealth ($ 000)	95.50	37.41	46.36	
	(94.46)	(110.47)	(148.83)	
Unemployment Rate	0.116	0.130	0.060	0.093
# of Unemployment Spells	231	104	645	
Fraction of Worker-Searcher			0.97	
Fraction of Searcher-Searcher			0.03	

We report here some additional details on the SIPP and on sample selection. The SIPP consists of a series of nationally representative panels of individuals who are followed for multiple years. Individuals are interviewed every four months. At each interview (wave), a retrospective employment, earnings, and benefits history for the last four months is collected.

To construct a sample of individuals with strong labor force attachment who are likely to engage in job search when out of work, we exclude individuals if: they are enrolled in school (part-time or full-time) during the course of the sample, own a business, report themselves as out of the labor force for more than quarter of the sample period, or have non-employment spells lasting more than 52 weeks. We define "couples" as family households who are continuously married and, similarly, "singles" as individuals living in non-family households who are continuously singles. Since ours is a two-state model of the labor market, we organize the data accordingly. We categorize an individual as employed or not employed (unemployed, thereafter) for each week of the sample to determine his/her labor market status. We then rearrange observations into an individual-spell format. We drop left-censored spells and unemployment spells shorter than one week, as many of them are associated with direct job-to-job transitions (Nagypal, 2008).

Table C1 reports some key summary statistics for our final sample.